Application of the Grade of Membership model (GoM) to delineate an Urban Hierarchy in Brazilian Amazonia[1]

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1 Introduction

The discussions on the urban nature in the Amazon and the urban nets in the region have raised important questions, which consider the peculiarities related to the urbanization process, spatial distribution and the intensity of material and immaterial flows. The studies on urban hierarchy have incorporated yet other relevant aspects, such as the correlations of size, power and competitiveness of the regional cities.

In balanced urban nets, the hierarchy of the cities, when properly planned and respected, is capable of bringing a series of economical and logistical benefits. The idea of an urban net connects to the existence of centers in a hierarchical distribution. Only in a hypothetical situation it is possible to imagine a region dominated by centers that are equally "sized" (demographically, functionally, among other aspects). In that way, it is plausible to imagine that, where there is a net, there is an urban hierarchy.

Throughout the 20th century, a series of academic studies was produced, in which the idea of a hierarchical organization of the cities appears implicitly or explicitly, basing itself on a few essential questions that have been leading theoretical and empirical efforts: why do cities present different population sizes? Is there any connection between the size and the growth of cities? How do economic activities respond to this differentiation in the demographical size of urban centers? How do economic activities create this differentiation? Is there, in fact, regularity in the population size distribution between the cities of a certain region? And if there is, why does this happen?

In an overall manner, it is possible to state that literature was influenced by two schools of thought. The first is supported by the Central Place Theory, elaborated by Christaller (1933) and improved by Lösch (1940). The second was developed with basis on the urban system model, elaborated by Henderson (1974) and (Krugman, 1996). The Central Place Theory takes into account that different population sizes create different conditions and opportunities for the growth of economic and functional activities. Later, Henderson (1974) establishes a model in which the optimal size of a certain city would be influenced, mostly, by the type of economic activity.

Other studies that have been elaborated are also worth mentioning, such as Zipf's in 1949, which claims the existence of an impressive empirical regularity in the distribution of urban population sizes, verified in several regions of the world. Furthermore, the model of random city growth, developed by Simon (1955), also deserves to be highlighted, since it was commonly quoted and discussed in the past decades.

More recently, the studies[2] produced within the school that is called the New Economic Geography (NEG) continue the debates, based on the idea that the scale refunds, relating to the city population growth, are not as constant as in Simon's (1955) model. The NEG takes into account that the city population growth would be a result of a junction between "centripetal" and "centrifugal" forces that stimulate the concentration of economic activities.

[2] See Krugman (1996).

In Brazil, a study by IBGE (2008) fills in a gap that has existed over the last years, related to the detailed study of the city influence areas and the hierarchical organization of central localities in the country. The IBGE then established the primary knots of the Brazilian urban nets in 2007 with the help of a great number of secondary information, seeking to identify the influence regions of these centers, starting from the interaction nets that connect the cities.

Facing the complexity of the information brought up and used by IBGE (2008), which also counts with a series of flow variables, the approach included in the next topic does not intend to elaborate a direct outline of the urban hierarchy with the help of the Grade of Membership (GoM) model that exceeds the one already carried out by IBGE. However, it focuses on supplying a few new elements that help understanding the organization of the Amazon's urban nets. As seen in the IBGE study, the model when applied is based on the general idea that all of these aspects are, directly, role players in the hierarchical organization of the cities, that is, that the "greatness" of a city and its hierarchical position in the net are not measured simply by the number of people residing there.

2 The Grade of Membership (GoM) model

The Grade of Membership (GoM) model is used in delineating profiles, based on a heterogeneous and multidimensional database, which allows identifying clusters and describing the differences among these (Woodbury *et al*., 1978; Woodbury & Manton, 1989; Manton *et al*., 1994; Cassidy *et al*., 2001).

In Brazil and abroad, the methodology has been widely used in elaborating analysis connected to the study of epidemics and health demography[3]. However, the method is not restricted to these research fields, since it is applicable to other studies with several other purposes. In this text, GoM will be used to broaden the possibilities of studying the urban hierarchy in the Legal Amazon.

Apart from most of cluster analysis statistic methods, GoM does not consider that people and objects are organized in well-defined groups. In GoM, a same individual (or observation) may have a certain pertinence degree to multiple groups, hence it is also being called a model of fuzzy sets (Machado, 1997). Furthermore, GoM has, among others, the quality of analyzing categorical data with small samples of a large number of variables.

According to Sawyer *et al.* (2002), this methodology applying in delineating profiles considers that: a) the unobserved association among the variable categories in the model delineates two or more well-determined profiles that are called extreme profiles; b) these extreme profiles have all the properties of classic closed sets; c) the pertinence degree to the extreme profiles are attributed to each individual. Thus, the individual that possesses all the characteristics of one of the extreme profiles will be 100% pertinent to that profile and 0% to the rest. The more this individual relates to the extreme profile, the more his pertinence level to this level increases. It is not unusual for them to be individuals that are equally distant to all the extreme profiles, not possessing, therefore, characteristics that relate them to the generated profiles. d) the pertinence degrees of the

[3] See Sawyer *et al* (2004); Alves *et al* (2008); Maetzel *et al* (2000); MacNamee (2004).

individuals form a fuzzy set and the bigger the number of variables, the more defined is the set; e) in GoM, as the elements in this set are individual attributes, the variety issue, included and badly handled in many statistical methods, is not a problem; f) the method parameters are estimated by iterative processes and, therefore, the smaller the sample, the smaller its convergence time (Sawyer *et al*., 2002).

According to Sawyer *et al*. (2002),

> "items (c) and (d) give the method, within reasonable limits, the benefit of better results, the smaller the size of the sample and the bigger the number of variables" (Sawyer *et al.*, 2002, p. 759).

The authors still state that,

> "as the pertinence degree of each individual is given by conjunction, in this individual, of all the variable categories in the model, the method shows, and in a very simple way, the variety included in the sample" (Sawyer *et al.*, 2002, p. 759).

The method demands the estimate of a pertinence degree score, for each individual, relative to the several sets, that is, the fuzzy division of the individuals, in order to obtain the extreme profiles. For each element in a fuzzy set there is a pertinence degree score (g_{ik}) that represents the level with which the element "i" belongs to the extreme profile k (Sawyer, 2000). These scores vary from 0 to 1: 0 indicates that the element does not belong to the set; 1 indicates that the element belongs entirely to the set. The g_{ik} represents the proportion or pertinence intensity to each extreme profile. Therefore, there are the following restrictions for the measure:

$$g_{ik} \geq 0 \quad \text{for each } i \text{ and } j$$

$$\sum_{k=1}^{k} g_{ik} = 1 \quad \text{for each } i$$

In order to form the model and the parameter estimate (scores), the following predispositions are necessary, according to Woodbury *et al.* (1978, p. 201):

a) "the random variables represented by Y_{ijl}, where "i" refers to the individual, "j" to the question and "l" to the answer category of each variable, are independent for each "i". That is, the answers to different individuals are independent;

b) the g_{ik} (k = 1, 2, …, k) are outcomes of the random vector components $\zeta_i = (\zeta_{il}, ..., \zeta_{ik})$ with a distribution function $H(x) = P(\zeta_i \leq x)$. That is, GoM scores are outcomes of random variables when an individual is selected in the population. The outcome sample distribution (or scores in the sample) gives estimates for the distribution function H(x);

c) if the pertinence degree g_{ik} is known, the "i" individual answers to the many Y_{ijl} questions are independent for each variable category;

d) the probability of "l" answer for the j^{th} question by an individual with the k^{th} extreme profile is λ_{kjl}. According to the

model assumption, there is at least one individual that is a well defined member of the k^{th} profile. This assumption gives the probability of answer for this individual to the several levels of this question. Then, one can write this assumption as being:

$$\lambda_{kjl} \geq 0 \quad \text{for each } k, j \text{ and } l$$

$$\sum_{k=1}^{k} \lambda_{kjl} = 1 \quad \text{for each } k \text{ and } j$$

e) the probability of a level "l" answer of the j^{th} question by the i^{th} individual, conditioned to the g_{ik} score will be given by:

$$P(Y_{ijl} = 1) = \sum_{k=1}^{k} g_{ik} \lambda_{kjl} = 1"$$

According to the assumptions above, the probability model for the construction of a maximum likelihood estimation procedure is formulated. The probability model for a random sample is the product of a multinomial model by each cell probability, given by:

$$E(Y_{ijl}) = \sum_{k=1}^{k} g_{ik} \lambda_{kjl}$$

where g_{ik} is, by assumption, known and greater than or equal to zero.

Considering the assumptions above, the maximum likelihood model can be written as:

$$L(y) = \prod_{i-1}^{I} \prod_{j=1}^{J} \prod_{l=1}^{L} (\sum_{k=1}^{k} g_{ik} \lambda_{kjl})^{yijl}$$

The software chosen to run the model is "GoM", freeware 3.3 version, developed by Peter Charpentier, from the Epidemiology and Public Health Department of the Yale University Medicine School, USA.

3 Applying GoM in the outline of urban hierarchy on the Legal Amazon

GoM uses several types of variables that aim for a better comprehension of the greatness and the influence capacity of Amazonian cities. To fulfill this task, a model that takes into account a variety of aspects which exceed the purely economic or demographic analysis is proposed. Thus, the variables which measure functionality and basic and specialized services offer capacity are very worthy, as well as the access to asset indicators and also those referring to equipment and infrastructure in the city.

To generate the analysis model for the present study, only the municipalities with a population higher than 20,000 inhabitants were taken into account. Even if the municipalities with a population size between 10,000 and 20,000 can take on a certain degree of importance concerning centrality in the Legal Amazon context, only those

with over 20,000 were considered, focusing the analysis on the spaces which hosted the biggest urban transformations in the region.

In this study, the scale chosen for the analysis is the municipal one. This is due to the available information, mostly regarding municipalities and not cities. Yet, there are many issues in the legal definitions of city and field in Brazil. Furthermore, the present study considers that, in many parts of the Amazon, the activities that are developed beyond the city urban perimeter often obey a logic far from being considered rural, minimizing the problems that may appear in this sort of approach.

CHART 1 presents the correlation of the variables present in the model, separated into six groups, according to the information nature, such as: spatial, demographic, socioeconomic, infrastructure and services, access to assets, functional. Whilst presenting the variables, the justification for its use will be followed by a descriptive analysis of information of the main variables that remained unexplored in the previous chapters.

Table 1: GoM model internal variable list

Variable nature	Variables
1 - Espacial	1.1. Centrality Indicator: variable that represents the number of times the city in question was verified as being the closer urban center and with a bigger population
2- Demographic	2.1. Municipality urbanization degree
	2.2. Municipality population in 2007
	2.3. Municipality MCT (Management Commitment Term) between 2000 and 2007
	2.3. Mesoregion MCT between 2000 and 2007
3- Socioeconomic	3.1. GNP (Gross National Product)
	3.2. Value of Municipality Participation Fund
	3.3. Proportion of poor people
	3.4. Municipality HDI (Human Development Index)
4- Infrastructure and services	4.1. % of people with access to treated water service
	4.2. % of people with access to electricity service
	4.3. % of people with access to garbage collection service
	4.4. Number of fundamental learning schools
	4.5. Number of medium level learning schools
	4.6. Number of fundamental learning enrolments
	4.7. Number of medium level learning enrolments
	4.8. Number of superior level learning enrolments
	4.9. Hospitals
	4.10. Hospital beds
	4.11. Health stations
	4.12. Health centers
5- Access to assets	5.1. Vehicle fleet
	5.2. % of people with computer
	5.3. % of people with television set
	5.4. % of people with refrigerator
	5.5. % of people with telephone
6- Functional	6.1. This variable is the result of a municipality functionality matrix in relation to total (37) of functions with several levels of specialization.

Source: Elaborated by the author

Embodying the spatial variables seems to be of great importance, since the urban hierarchy is also defined under the influence of the city distribution in the net, with evident impacts in how they interact and relate to each other. Variable 1.1 represents the number of times which a certain city was seen to be the closest urban center and being of the greatest population size. Each time that a city is seen as the biggest and closest to any of the 20,000-inhabitant centers (2007) in the Legal Amazon, the city gains 1 point

in the so-called Centrality Indicator. Furthermore, the Centrality Indicator also accumulates points from the city relations to belonging centers up to the seventh order in the net, with differentiated values (0.5 for the second order, 0.25 for the third, 0.125 for the fourth and so on).

The four obtained profiles were selected based on ten generated results with random initial λ_{kjl} , that is, ten models were generated from four profiles. The constancy observed in the final λ_{kjl} obtained in the ten models indicated that the global maximum (mathematical criteria for optimization) was duly achieved in all the models. One out of the ten models was chosen based on the coherence of the results found for the municipal population in 2007 which, without the slightest doubt, is one of the main variables regarding the comprehension of the urban hierarchy[4]. Thus, although GoM was not exclusively developed for delineating hierarchy patterns, this emerged naturally from the information, revealing profiles that matched the expectations.

The profile description and denomination were effected based on the ratio between each expected probability (E) in the level (l) of the variable (j) in the extreme profile (k), that is, λ_{kjl}, and the observed probability (O) of the answer (l) of the variable (j) for any municipality (marginal probabilities). This ratio can be denominated, in a simplified manner, as (E/O). A ratio E/O superior to 1.2 is an indication that the profile has a "remarkable" or "descriptive" characteristic; this criterion is proposed by Sawyer *et al*. (2002).

The following profile description is made according to the expected probability (E) of each variable level relative to the observed marginal probability (O). That is, the profiles are described based on the characteristics with an E/O ratio equal or superior to 1.2, as seen before. It is important to notice that this description is referring to the pure types (g_{ik} =1) of each profile.

Profile 1: 1) high urbanization degree (2000), superior to 80%; 2) average to high size population (2007), greater than 50,000 inhabitants; 3) high municipal MCT (2000-2007), between 3 and 6% a year; 4) small/average positive mesoregion MCT (2000-2007), between 0.5 and 1% a year and between 1.5 and 2% a year; 5) average to high GNP (2005), greater than R$500,000,000 until the class R$12,000,000,000 or more; 6) average/high MPF (2005), greater than R$8,000,000 until the class R$100,000,000 or more; 7) relatively small proportion of poor people (2000) for the regional pattern, less than 45%; 8) average and high HDI (2000), between 0.71 and 0.80 or more; 9) average/high proportion of people with access to treated water service (2000), superior to 60%; 10) very high proportion of people with access to electricity service (2000), superior to 90%; 11) high proportion of people living in urban houses with garbage collection service (2000), superior to 80%; 12) high number of fundamental learning schools (2006), greater than 101 and including the class of 601 or more; 13) average/high number of medium level schools (2006), superior to 8.11 and including the class 61.07 or more; 14) average/high number of enrolments in fundamental learning (2006), superior to 20,001 and including the class 250,001 or more; 15) average/high number of enrolments in medium level learning (2006), superior to 2,501; 16) average/high number of enrolments in superior level learning (2007), greater than

[4] It is important to note that this selection procedure of the most adequate model is explained by Manton et al (1994).

1,001 and including the class 40,001 or more; 17) high number of hospitals (2000), superior to 4 and 5 or more; 18) average/high number of hospital beds (2000), greater than 101 and including the class 3,601 or more; 19) average/high number of health stations (2000), superior to 11 and including the class 51 or more; 20) average/high number of health centers, superior to 3 and including the class 33 or more; 21) average/high vehicle fleet (2007), superior to 15,001 and including the class 250,001 or more; 22) average/high proportion of people living in houses with computer (2000) for the regional patterns, superior to 3% and including the class 10.01% or more; 23) high proportion of people living in houses with electricity service and television (2000), superior to 80.01%; 24) high proportion of people living in houses with electricity service and refrigerator (2000), superior to 80.01%; 25) high proportion of people living in houses with telephone (2000) for the regional patterns, superior to 30%; 26) average/high/very high centrality indicator, superior to 2.51 and including the class 30.01 or more; 27) high/average functional diversification, presenting more than 60.01% of the functionalities.

Profile 2: 1) average to high urbanization degree (2000), predominantly between 70 to 90%; 2) average size population (2007), between 30,000 and 100,000, with bigger emphasis to the municipalities with more than 50,000 inhabitants; 3) very high municipal MCT (2000-2007), greater than 6% a year, or negative, between -2.99 and -1.5% a year; 4) high positive mesoregion MCT (2000-2007), between 2.5 and 3% a year and greater than 3% a year or small positive, less than 0.5% a year; 5) average/small GNP (2005), between R$500,000,000 and R$1,500,000,000 and less than R$500,000; 6) average/small MPF (2005), between R$8,000,000 and R$16,000,000 and less than R$8,000,000; 7) average proportion of poor people (2000) for the regional pattern, between 27% and 58.51%; 8) average HDI (2000), from 0.71 to 0.8; 9) average proportion of people with access to treated water service (2000), between 40% and 80%; 10) high proportion of people with access to electricity service (2000), superior to 80%; 11) average proportion of people living in urban houses with garbage collection service (2000), less than 60%; 12) small number of fundamental learning schools (2006), less than 50; 13) small number of medium level schools (2006), inferior to 8.11; 14) small number of enrolments in fundamental learning (2000), between 5,001 and 7,500; 15) average number of enrolments in medium level learning, between 1,001 and 5,000; 16) small/average number of enrolments in superior level learning (2007), between 1 and 1,000; 17) average number of hospitals (2000), between 2 and 4; 18) average number of hospital beds (2000), between 101 and 400; 19) average number of health stations (2000), between 6 and 10; 20) average number of health centers, between 2 and 8 and between 17 and 32; 21) average vehicle fleet (2007), between 5,001 and 15,000; 22) average proportion of people living in houses with computer (2000) for the regional patterns, between 2% and 5%; 23) average/high proportion of people living in houses with electricity service and television (2000), between 60,01% and 90%; 24) high proportion of people living in houses with electricity service and refrigerator (2000), between 60,01 and 80%; 25) average proportion of people living in houses with telephone (2000) for the regional patterns, between 10.01% and 30%; 26) small centrality indicator, between 0.01 and 2.5; 27) average functional diversification, presenting between 40.01% and 70.01% of the functionalities.

Profile 3: 1) average urbanization degree (2000), between 50 to 70%; 2) average size population (2007), from 30,000 to 100,000, with bigger emphasis to the municipalities with population less than 50,000 inhabitants; 3) negative municipal MCT (2000-2007),

between -1.5 and 0% a year; 4) moderate positive mesoregion MCT (2000-2007), between 0.5 and 1% a year and between 1% and 1.5% a year; 5) small GNP (2005), less than R$500,000,000; 6) average/small MPF (2005), between R$8,000,000 and R$16,000,000 and less than R$8,000,000,000; 7) high proportion of poor people (2000) for the regional pattern, between 58.52% and 79.59%; 8) small HDI (2000), from 0,61 to 7; 9) small proportion of people with access to treated water service (2000), between 20% and 40%; 10) average proportion of people with access to electricity service (2000), between 60% to 90%; 11) average/small proportion of people living in urban houses with garbage collection service (2000), between 10% and 70%; 12) average/small number of fundamental learning schools (2006), between 50 and 200; 13) small number of medium level schools (2006), inferior to 8.11; 14) average/small number of enrolments in fundamental learning (2000), between 7,501 and 20,000; 15) average number of enrolments in medium level learning, between 1,001 and 5,000; 16) small/average number of enrolments in superior level learning (2007), between 1 and 1,000; 17) small/average number of hospitals (2000), 1, 2 or 4; 18) average number of hospital beds (2000), between 101 and 400; 19) average/high number of health stations (2000), between 6 and 10 and superior to 31; 20) average number of health centers, between 3 and 4; 21) small vehicle fleet (2007), equal to or less than 5,000; 22) small proportion of people living in houses with computer (2000) for the regional patterns, between 1% and 2%; 23) average proportion of people living in houses with electricity service and television (2000), between 50.01% and 70%; 24) average proportion of people living in houses with electricity service and refrigerator (2000), between 40.01 and 60%; 25) small proportion of people living in houses with telephone (2000) for the regional patterns, between 5.01% and 10%; 26) average centrality indicator, between 2.51 and 5; 27) small functional diversification, presenting between 30.01% and 50.01% of the functionalities.

Profile 4: 1) average to high urbanization degree (2000), between 10 to 50%; 2) small size population (2007), from 20,000 to 30,000 inhabitants; 3) very high municipal MCT (2000-2007), between 3 and 6% a year and greater than 6% a year, or very small MCT, less than -3% a year; 4) average positive and moderately high mesoregion MCT (2000-2007), between 1 and 1.5 % a year and between 2% and 3% a year; 5) small GNP (2005), less than R$500,000; 6) small MPF (2005), less than R$8,000,000; 7) high proportion of poor people (2000) for the regional pattern, between 64.92% and 72.29% and very high, between 72.305 until the class between 79.6% or more; 8) very small HDI (2000), less than 0.6; 9) very small proportion of people with access to treated water service (2000), less than 20%; 10) average proportion of people with access to electricity service (2000), less than 60%; 11) small proportion of people living in urban houses with garbage collection service (2000), less than 60%; 12) small number of fundamental learning schools (2006), between 50-100; 13) small number of medium level schools (2006), inferior to 8.11; 14) small number of enrolments in fundamental learning (2000), less than 10,000; 15) small number of enrolments in medium level learning, less than or equal to 1,000; 16) there is no enrolments in superior level learning (2007), between 1 and 1,000; 17) small number of or no hospitals (2000), equal to 1 or 0; 18) small number of hospital beds (2000), less than 100; 19) small/average number of health stations (2000), between 11 and 20 and less than or equal to 5; 20) small number of health centers, equal to or less than 1; 21) small vehicle fleet (2007), equal to or less than 5,000; 22) very small proportion of people living in houses with computer (2000) for the regional patterns, less than or equal to 1%; 23) small proportion of people living in houses with electricity service and television (2000), less than 50%;

24) small proportion of people living in houses with electricity service and refrigerator (2000), less than 40.00%; 25) very small proportion of people living in houses with telephone (2000) for the regional patterns, less than 5.00%; 26) centrality indicator equal to 0; 27) very small functional diversification, presenting less than 40% of the functionalities.

Complementary to these descriptions, CHART 2 presents the $g_{ik(s)}$ distribution in the model four profiles. The fact that, in all the profiles, 138 municipalities (57%) had a high degree of compatibility with $g_{ik(s)}$, superior to 0.75 (which is considered very high), is another indicative which validates the profile numbers that were found and the model adequacy to the present study data (that is, the profiles "fit" adequately to most of the municipalities). Furthermore, 97 municipalities (40%) had $g_{ik(s)}$ between 0.51 and 0.75 (considered high). Thus, 97% of the municipalities have $g_{ik(s)}$ with amounts higher than 0.50 in one of the profiles, which is quite interesting, since almost all the municipalities have a high compatibility degree to some profile. It is worthy to note that a municipality with a minimum of 0.51 in one profile cannot have a pertinence superior to 0.49 in any other profile.

Table 2 – $g_{ik(s)}$ distribution

Profile 1			
	Frequency	**%**	**Accumulated %**
0 – 0.25	193	79.75	79.75
0.26 – 0.50	22	9.09	88.84
0.51 – 0.75	5	2.07	90.91
0.76 +	22	9.09	100.00
Total	242	100.00	

Profile 2			
	Frequency	**%**	**Accumulated %**
0 – 0.25	164	67.77	67.77
0.26 – 0.50	20	8.26	76.03
0.51 – 0.75	22	13.22	89.26
0.76 +	26	10.74	100.00
Total	242	100.00	

Profile 3			
	Frequency	**%**	**Accumulated %**
0 – 0.25	137	56.61	56.61
0.26 – 0.50	39	16.12	72.73
0.51 – 0.75	33	13.64	86.36
0.76 +	33	13.64	100.00
Total	242	100.00	

Profile 4			
	Frequency	**%**	**Accumulated %**
0 – 0.25	136	56.20	56.20
0.26 – 0.50	22	9.09	65.29
0.51 – 0.75	27	11.16	76.45
0.76 +	57	23.55	100.00
Total	242	100.00	

Source: Elaborated by the author

According to Figure 1, it is clear that the municipalities with a high compatibility to Profile 1 are distributed along the main roads in the Legal Amazon, with a highlight to those between Cuiabá and Rio Branco, in the outskirts of BR 364 and BR 070: Cuiabá, Barra do Garças, Rondonópolis, Várzea Grande, Tangará da Serra, Vilhena, Ji-Paraná, Ariquemes, Porto Velho and Rio Branco. One can also find some other municipalities with a g_{ik} higher than 0.75 in the proximities of the Belém – Brasília highway, for example: Gurupi, Palmas, Araguaína, Imperatriz, Castanhal and Ananindeua. Far from each other on the map, some state capitals stand out, such as Manaus, Macapá, São Luís and Boa Vista. Out of this group of municipalities, only Sinop, in highway 163, seems more far and out of line with the medium/large agglomerations of the region. All the state capitals reached maximum compatibility with profile 1, whereas seven intermediate-sized municipalities also achieved this amount[5].

Profiles 2 and 3, in an overall manner, present characteristics seen as intermediate in relation to the other profiles. It is worth mentioning that Profiles 2 and 3 present very distinct spatial patterns. When it comes to the municipalities with g_{ik} higher than 0.75 in Profile 2, Figure 2 makes it clear that these are concentrated in the inner region of Mato Grosso and in the eastern portion of the Legal Amazon. As for the municipalities with high compatibility rates to Profile 3, they are located in the inner region of the Amazonas, Acre, and in the western part of Pará and Maranhão.

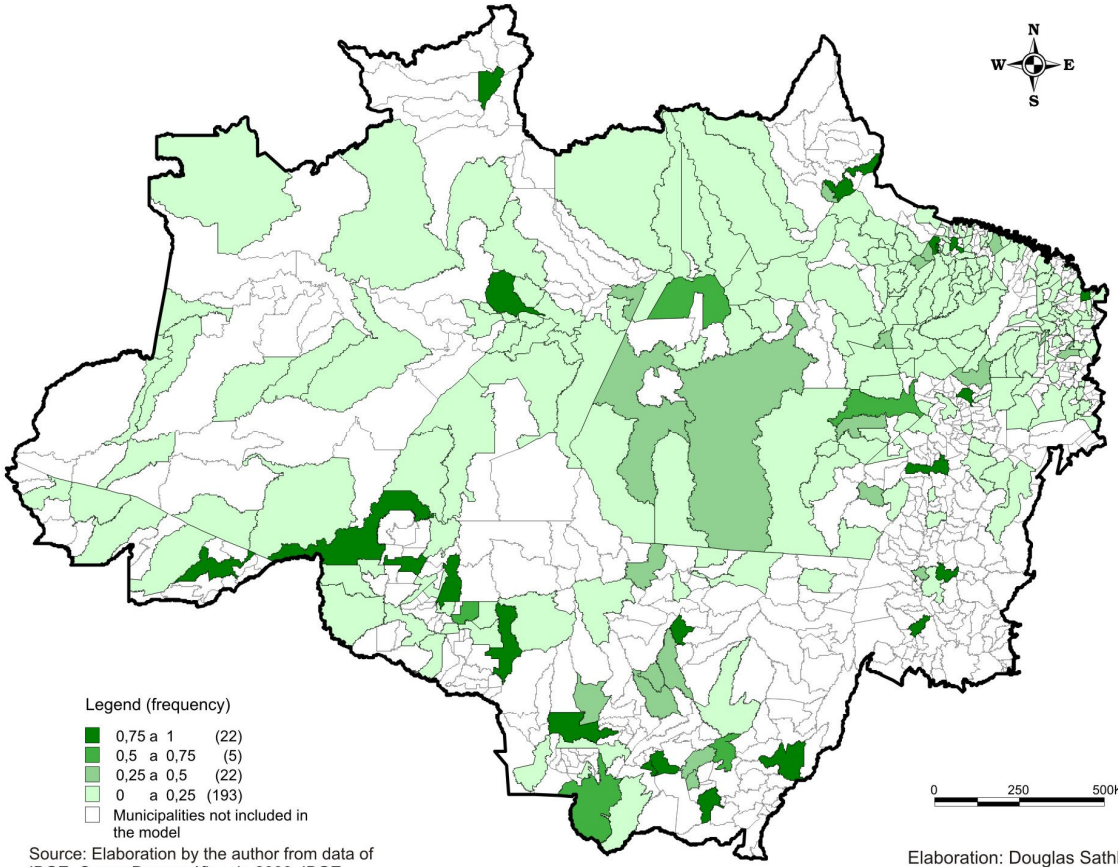
The municipalities that belong to Profiles 2 and 3 are demographically medium-sized. It is clear that Profile 2 municipalities, more urbanized and more populated than Profile 3, are those placed by the highways and roads. Profile 3 municipalities are preferably located along the margins of the main rivers that cross the inner region. Figures 2 and 3 seem to suggest that the highway influence contributed, more than the traditional transportation means in the region, to this size differentiation, urbanization degree, and functional diversity, among others.

Profile 4 municipalities with elevated $g_{ik(s)}$ are concentrated in the inner regions of Pará and Maranhão, and, on a smaller scale, in Amazonas, Acre and Rondônia (Figure 4). The municipalities with a high compatibility to Profile 4 were the ones with more pure types (30) in relation to the other profiles.

GoM applying seems to suggest that the use of strictly demographic criteria would be capable of delimitating, with a certain amount of efficiency, the urban hierarchical levels, since, in the Legal Amazon, many of the variables (socioeconomic, infrastructure and services indicators, access to assets and functional diversity) are positively correlated with the population size of the municipalities. That is, the less populated municipalities with high compatibility rates to Profile 4 are also those with the worst socioeconomic indicators and the biggest needs of the basic service offer, as well as low access to assets. The intermediate-sized municipalities, with high compatibility rates to Profiles 3 and 2, seem to be in a more favorable situation than those in Profile 4. The municipalities with a high compatibility rate to Profile 1, with medium/large population sizes, are those that offer the "best" socioeconomic indicators in the Region.

[5] Tangará da Serra, Rondonópolis, Barra do Garças, Imperatriz, Araguaína, Ananindeua, and Ji-Paraná.

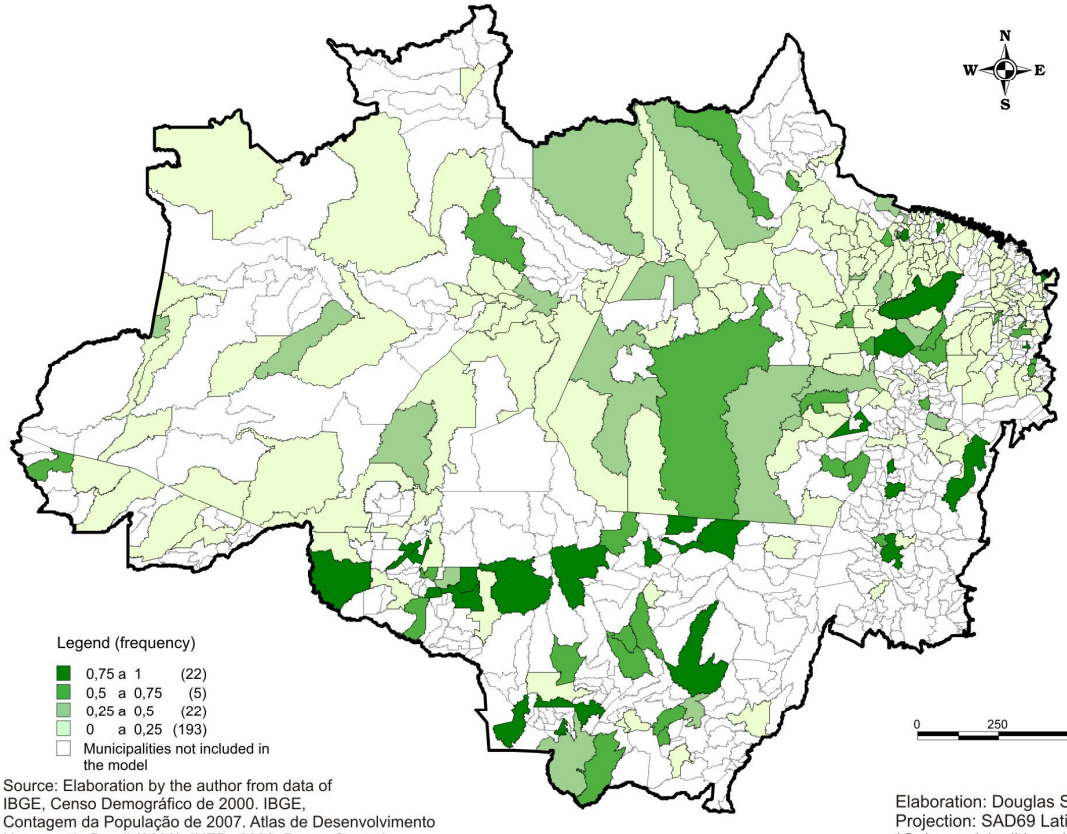
Figure 1 - The Legal Amazon: Distribution of the g1 values of the municipalities



Source: Elaboration by the author from data of IBGE, Censo Demográfico de 2000. IBGE, Contagem da População de 2007. Atlas de Desenvolvimento Humano do Brasil (2000), INEP, 2006, Banco Central 2004, IBGE, Estatísticas de Saúde, 2000. IBGE, Perfil dos Municípios - Cultura - 2006.

Elaboration: Douglas Sathler
Projection: SAD69 Latitude x Longitude
*Only municipalities with more than 20.000 inhabitants

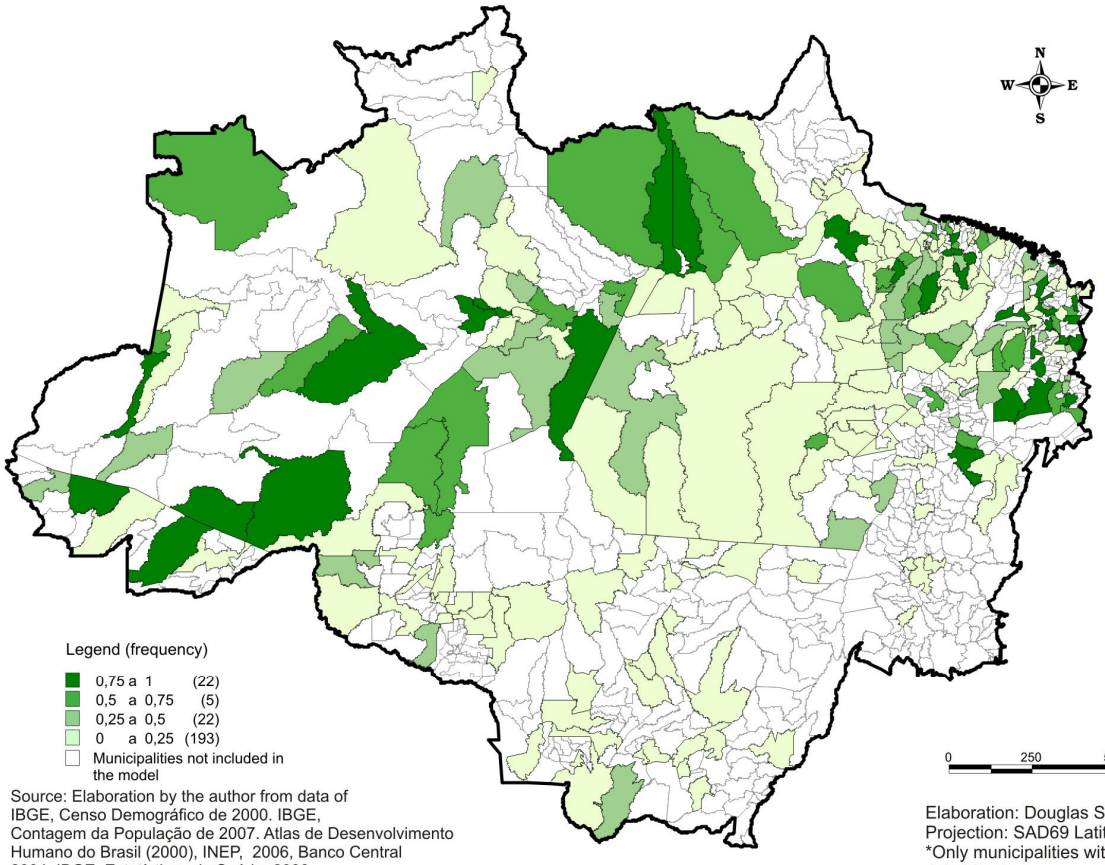
Figure 2 - The Legal Amazon: Distribution of the g2 values of the municipalities



Source: Elaboration by the author from data of IBGE, Censo Demográfico de 2000. IBGE, Contagem da População de 2007. Atlas de Desenvolvimento Humano do Brasil (2000), INEP, 2006, Banco Central 2004, IBGE, Estatísticas de Saúde, 2000. IBGE, Perfil dos Municípios - Cultura - 2006.

Elaboration: Douglas Sathler
Projection: SAD69 Latitude x Longitude
*Only municipalities with more than 20.000 inhabitants

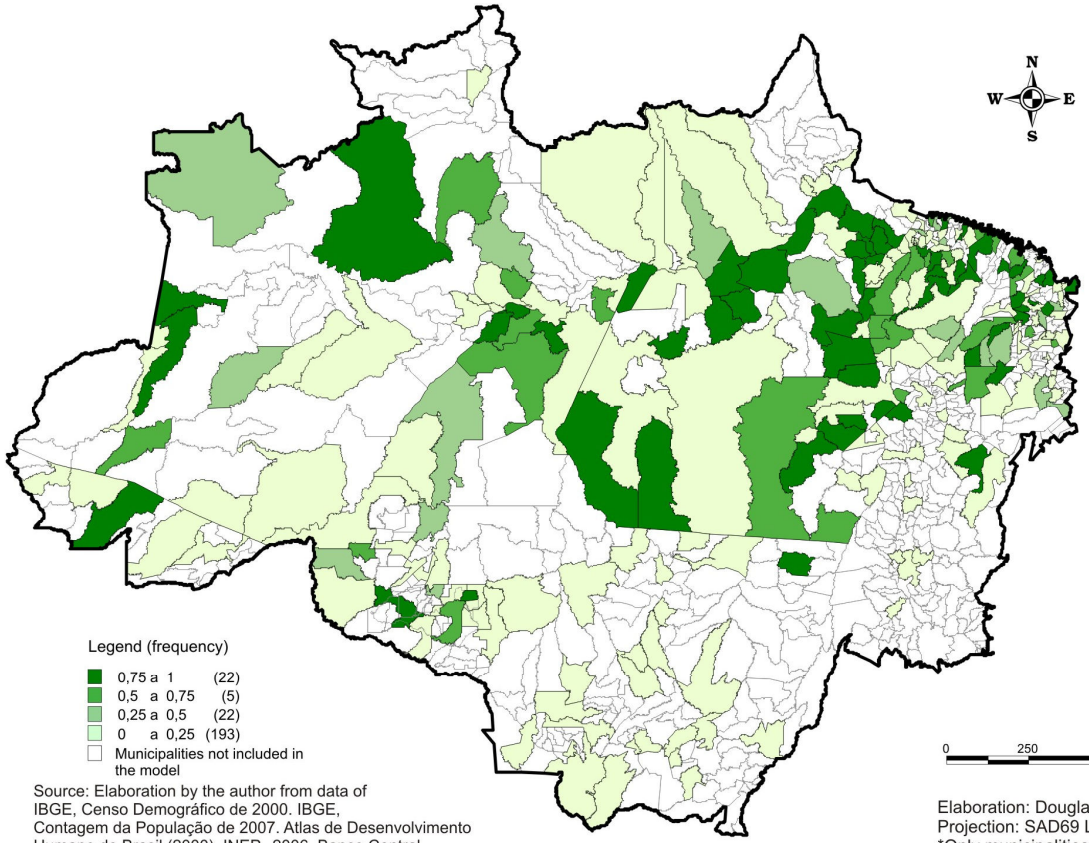
Figure 3 - The Legal Amazon: Distribution of the g3 values of the municipalities



Source: Elaboration by the author from data of IBGE, Censo Demográfico de 2000. IBGE, Contagem da População de 2007. Atlas de Desenvolvimento Humano do Brasil (2000), INEP, 2006, Banco Central 2004, IBGE, Estatísticas de Saúde, 2000. IBGE, Perfil dos Municípios - Cultura - 2006.

Elaboration: Douglas Sathler
Projection: SAD69 Latitude x Longitude
*Only municipalities with more than 20.000 inhabitants

Figure 4 - The Legal Amazon: Distribution of the g4 values of the municipalities



Source: Elaboration by the author from data of IBGE, Censo Demográfico de 2000. IBGE, Contagem da População de 2007. Atlas de Desenvolvimento Humano do Brasil (2000), INEP, 2006, Banco Central 2004, IBGE, Estatísticas de Saúde, 2000. IBGE, Perfil dos Municípios - Cultura - 2006.

Elaboration: Douglas Sathler
Projection: SAD69 Latitude x Longitude
*Only municipalities with more than 20.000 inhabitants

However, the model showed some interesting results that escape from this general tendency. In Santarém's case, the biggest city in the inner region of the Amazon (non-capital), the compatibility rate to Profile 1 was relatively low (0.65 to Profile 1), considering its demographic size. Municipalities with less than half its population, such as Ji-Paraná and Araguaína, appear as pure Profile 1 types, amounts that are well above Santarém's. Marabá (0.57) with almost 200,000 inhabitants, Itaituba (0.25), Abaetuba (0.36), Parauapebas (0.43) and Parintins (0.28), all with population sizes superior to 100,000 inhabitants, also present low compatibility rates to Profile 1, considering the demographic size of these municipalities.

Some of the model variables caused many medium-sized municipalities to be included as pure Profile 1 types along with big municipalities, such as São Luís, Belém and Manaus. This seems to make sense in some variables. In order to have a general idea, the level of some medium-sized municipal functional diversity is very close to what was verified for the largest cities of the region. Besides, the variables that measure percentage, proportion and degree also contribute to this result.

4 Conclusion

In what regards to the organization in hierarchical levels between the cities of the Amazon's urban nets, theoretical pond rations suggest that the centrality condition handed over to a certain city is related to an association of qualities and characteristics. So, it is often the case that the studies that sought to understand the hierarchical organization of the cities were closely linked to the demographical size and to the shape in which this variable influenced the economic variables and the functions of urban agglomerations or vice-versa. Being so, it is possible to state that the urban hierarchy is not to be adequately evaluated focusing merely on the demographical size of the centers, or even on the way the population size of a city is affected by economic variables. Even when it comes to the Legal Amazon, in which, as seen before, generally there is regularity in population sizes with variables of a different kind in the delimitation of hierarchical patterns, it is noticeable that some municipalities seem to escape from this tendency.

GoM model showed that a municipality in the inner region, with a high degree of compatibility with Profile 1, that is, with a g_{ik} higher than 0.75, is more likely to contain a centrality that plays a functional role closer to what is understood as a "medium city", considering all the conceptual complexity inlaid in the term. Thus, 23 municipalities presented a high g_{ik} in Profile 1. They include all the state capitals of the Legal Amazon, which were qualified as being pure Profile 1 types. Considering that a state capital, even a demographically middle-sized one, is generally on the top of the regional hierarchy and, therefore, would not be qualified as a medium city, it is noticeable that, in this discussion applied to the Amazon, the municipalities of Ji-Paraná, Araguaína, Imperatriz, Barra do Garças, Rondonópolis, Tangará da Serra, Várzea Grande, Ariquemes, Sinop, Gurupi, Castanhal and Vilhena are highlighted due to the high compatibility with Profile 1, which is characterized by the medium/large population size, high degree of urbanization, high functional diversification and medium/high GDP (Gross Domestic Product).

GoM demonstrated that some municipalities that contain cities of expressive population contingent in the inner region of the Amazon (not the capitals) do not present a high

compatibility with Profile 1, which denounces the existing needs in part of the medium sized municipalities in the region, such as in Santarém, the biggest city in the inner region of the Amazon, Marabá, Itaituba, Parauapebas, Abaetetuba and Parintins.

GoM also permits evaluating the existence of differentiated patterns in the locality influence (highway – border areas / rivers – countryside) regarding to the model variables, since the description and spatialization of the municipalities with a high compatibility with Profiles 2 and 3 show this very clearly.

Profile 2, characterized by its medium to high urbanization degree (between 70% and 90%), by medium-sized population (30,000 to 100,000) and low to medium GDP (from R$ 500,000,000 to R$ 1,500,000,000), encloses a group of municipalities found predominantly in the "highway arch" that cuts through all the southern portion of the region. Profile 3 is characterized by a medium urbanization degree (from 50% to 70%), by a medium-sized population (30,000 to 100,000 people) and GDP lower than R$ 500,000,000, encloses municipalities located in a dispersed manner through the forest, and, mainly, near the main rivers in the region. It is noticeable that, in this case, the population, when analyzed in isolation, does not help to differentiate Profile 1 from 2.

Profile 4 is characterized by small municipalities that are little urbanized (from 10 to 50%), by a small-sized population (between 20,000 and 30,000 inhabitants), by a GDP lower than R$ 50,000,000 and by high proportion of poor people in the year 2000, between 64.92% and 79.6%. The localization of these municipalities does not follow the main road outlines of the region, but instead they are found near rivers and secondary roads, especially in the states of Pará and Maranhão.

Furthermore, GoM used two variables that aimed to provide more security to the model: the Functionality Indicator and the Centrality Indicator. With this methodological novelty, the analysis of the hierarchical patterns represented in the model by the 4 profiles were closer to what could be understood as a "methodological ideal", difficult to be applied empirically due to the complexity of the subject, but included in studies theoretically biased.

Understanding the hierarchical organization of the cities in Amazonia seems to be a very important exercise in order to comprehend the dynamics and specific characteristics of the regional urban nets. In this way, it is evident that policies which stimulate the establishment of more structured urban nets in the Amazon are needed. A more balanced population distribution throughout the territory could bring a series of benefits, especially when it comes to the offer and access to all different sorts of services.

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