1. Introduction

The age-specific first-marriage distribution has a typical bell shaped pattern common in all human populations with differences in mode, variability and levels. In order to estimate this pattern a number of parametric models have been proposed. Among them, the Coale-McNeil model (Coale, 1971; Coale and McNeil, 1972) is considered as a standard demographic tool for estimation and projection purposes (Bloom, 1982; Bloom and Bennett, 1990; Goldstein and Kenney, 2001). This is a relational model that estimates the density function of the distribution of women who ever marry by age at marriage using a nuptiality standard based on Swedish empirical data from the period 1865 to 1869. However, several studies show that this standard first marriages schedule derived from the Swedish experience may be inappropriate for some populations and the use of more adequate representations is required. To this end, Kaneko (2003; 2005) suggested the use of the Generalised Log Gamma model (GLM) that is mathematically identical to the Coale-McNeil one and also an improved adjusted version of it, in order to estimate the age pattern of first marriages and births. In addition, Liang (2000) suggested that in some populations, people who marry come from two different groups and therefore he proposed a mixture of the Coale-McNeil model.

In this paper an alternative parametric model that estimates the pattern of first-marriages of modern populations is presented. Three different versions of this model are provided, in order to capture the nuptiality pattern of both homogeneous and heterogeneous populations. For the evaluation of the efficiency of this model we fit it in a variety of empirical data sets of several European populations. For comparison reasons we also fit to the same data sets the various models proposed before by Coale-McNeil, Kaneko and Liang.

In Section 2 a review of the existing literature in the subject as well as a short presentation of existing models for fitting first-marriages data is provided. Then, in Section 3 the new model is presented. Section 4 provides an evaluation of the model proposed. Finally some remarks are given in Section 5.

2. Nuptiality models

Coale (1971) was the first who observed that age distributions of first marriages are structurally similar in different populations. These distributions tend to be smooth, unimodal, skewed to the right and have density close to zero below age 15 and above age 50. In addition he observed that the differences in age at first marriage distributions across female populations present differences in modes, standard deviations and tails. Moreover he showed that the age pattern of first marriage frequencies was virtually identical in populations characterised by widely different marriage customs when adjusting the vertical and horizontal scales and the origin of the distribution of first marriage frequencies.

In recent years a considerable variation is observed in the pattern of first-marriage in data sets for populations of several countries. Liang (2000) observed a bulge at the early ages, in first marriage rates of Chinese women. The new pattern of age-specific first-marriage curves reflects heterogeneity in its behaviour. Liang (2000) made the plausible hypothesis that observations may come from two different groups. This heterogeneity may be associated with many factors such as the educational level and the social status of the population as well as religion, or existence of population groups with different demographic characteristics regarding marriage but all these hypotheses need further investigation of the empirical evidence.

A variety of mathematical models have been proposed in the literature for fitting the first-marriage curve. However, not much work has been done in graduating it, when the bulge at early ages of the first-marriage curve is appeared.

Among the various models used for estimating the age-specific first-marriage pattern of populations several have been proved to provide accurate fits to the one-year specific first marriage rates. These are the Standard Coale-McNeil model, the Coale-McNeil model in his flexible form, the generalised log gamma model and the mixture Coale-McNeil model.

The Coale-McNeil (1972) model based on the standard schedule of first marriage frequencies using data from Sweden covering the period 1865-1869, proposed by Coale (1971) is expressed by,

$$f(a) = \frac{C}{k} g_s\left(\frac{a-a_o}{k}\right),$$

where $f(\alpha)$ is the first-marriage frequency at age α ; C denotes the proportion of people eventually married in the population; a_o is the origin of the observed distribution or equivalently the youngest age at which an appreciable number of first marriages occurs and which approximately is the first percentile of the distribution; k is the inverse of the rate at which first marriages occur in the observed population relative to the Swedish standard and $g_s(x)$ is the Coale-McNeil standard schedule, which is a closed-form expression that closely replicates the reference distribution presented by Coale (1971), developed by Coale and McNeil (1972) and expressed as,

$$f_s(x) = 0.1946\, exp\{-0.174(x-6.06) - exp[-0.2881(x-6.06]\}.$$

In a more general sense Coale and McNeil define the probability density function for the age distribution of first-marriages as,

$$f(x) = \frac{\beta}{\Gamma\left(a/\beta\right)} exp[-a(x-\mu-exp\{-\beta(x-\mu)\})] \ ,$$

where Γ denotes the gamma function and $a > 0, \beta > 0, -\infty < \mu < \infty$ are the parameters to be estimated.

The generalized log gamma model, proposed by Kaneko (1991, 2003) is expressed by,

$$f(x; C, u, b, \lambda) = C\frac{|\lambda|}{b\Gamma(\lambda^{-2})}\left(\lambda^{-2}\right)^{\lambda^{-2}} exp\left[\lambda^{-1}\left(\frac{x-u}{b}\right) - \lambda^{-2} exp\left\{\lambda\left(\frac{x-u}{b}\right)\right\}\right],$$

where $f(x)$ is the age specific first marriage rate at age x, C denotes the proportion eventually being married, $\lambda(-\infty < \lambda < 0), u(-\infty < u < \infty), b(> 0)$ are the three parameters to be estimated and Γ denotes the gamma function. In this model the Coale-McNeil distribution is expressed in PDF form of the GLM distribution according to the Prentice's parameterization (1974) given that the Coale-McNeil distribution is mathematically identical to the generalised log gamma distribution with a somewhat different parameter space.

The models described so far, cannot capture the first-marriage pattern that appears in some modern populations. Therefore a new model is required that takes into account the feature of the enhanced early age first-marriage. Liang (2000) based on the idea of Chandola et al. (1999) built a mixture model using the double-exponential distribution. This model denoted as the mixture Coale-McNeil model, is described,

$$f(x; m, \alpha_1, \lambda_1, \mu_1, \alpha_2, \lambda_2, \mu_2) = \frac{m\lambda_1}{\Gamma\left(\frac{\alpha_1}{\lambda_1}\right)} \exp(-\alpha_1(x-\mu_1) - e^{-\lambda_1(x-\mu_1)}) + \frac{(1-m)\lambda_2}{\Gamma\left(\frac{\alpha_2}{\lambda_2}\right)} \exp\left(-\alpha_2(x-\mu_2) - e^{-\lambda_2(x-\mu_2)}\right)$$

where m is the mixture parameter that determines the relative size of the group of women that marry at an earlier age.

3. The new model

Three versions of a parametric model for fitting the age-specific distributions of first-marriages are proposed.

The simple model is defined by the equation,

$$f(x) = c_1 \exp\left[-\left(\frac{x-\mu}{\sigma\ (x)} \right)^2 \right]$$

where *f(x)* is the age-specific first marriage rate at age *x*, $\sigma(x) = \sigma_{11}$ *if* $x <= \mu$, *while* $\sigma(x) = \sigma_{12}$ *if* $x > \mu$, and c_1, μ, σ_{11} *and* σ_{12} are parameters to be estimated.

The parameter c_1 is related to the base level of the distribution, μ reflects the location of the distribution, while σ_{11}, σ_{12} reflect the spread of the distribution before and after its peak, respectively.

Then, following the idea of Chandola et al. (1999) and Liang (2000), we propose a model with two terms (hereafter denoted as Mixture Model 1) in order to fit the first-marriage rates in heterogeneous populations. This model is given by the formula,

$$f(x) = c_1 \exp\left[-\left(\frac{x-\mu_1}{\sigma_1} \right)^2 \right] + c_2 \exp\left[-\left(\frac{x-\mu_2}{\sigma_2} \right)^2 \right]$$

where *f(x)* is the age-specific first marriage rate at age x, and $c_1, c_2\ \mu_1, \mu_2\ \sigma_1, \sigma_2$ are parameters to be estimated. The parameters c_1, c_2 are related with the base level of the first and the second hump respectively, μ_1, μ_2 reflect the location of the first and the second hump respectively. These are related to the modal ages of the two subpopulations, the one that gets married earlier and the second that gets married at later ages, while σ_{11}, σ_{12} reflect the variances of the two humps.

In some data sets the first-marriage curve is steeper in the left part of the first hump. This feature led us to make an adjustment to Mixture model 1 allowing an extra parameter to be imposed in the first term of the model. The new formula (hereafter Mixture model 2) thus becomes,

$$f(x) = c_1 \exp\left[-\left(\frac{x-\mu_1}{\sigma_1(x)} \right)^2 \right] + c_2 \exp\left[-\left(\frac{x-\mu_2}{\sigma_2} \right)^2 \right]$$

where $\sigma_1(x) = \sigma_{11}$ *if* $x <= \mu_1$, *and* $\sigma_1(x) = \sigma_{12}$ *if* $x > \mu_1$.

The parameters have the same meaning as in Mixture model 1, while σ_{11}, σ_{12} reflect the spread of the first hump before and after its peak respectively.

The parameters of the model have been estimated by means of a least-squares procedure by minimising the following sum of squares.

$$\sum_x \left(\widehat{f}_x - f_x\right)^2$$

where $\widehat{f}_x$ is the estimated first-marriage rate at age x and f_x is the empirical one.

It is generally accepted that for graduation purposes where it is important to bring out the structure of the underlying 'true' curves by removing the effects of random variations, weighted least squares with weights equals to the reciprocals of the estimated variances of the ungraduated rates is the most efficient curve fitting procedure (Hoem, 1976).

Therefore in our evaluations, we also tested weighted non-linear least squares estimation of the parameters of the model by minimising the following sum of squares:

$$\sum_x w_x \left(\hat{f}_x - f_x\right)^2$$

with weights w_x the reciprocals of the estimated variances of the age-specific first-marriage rates, i.e. the weights are given by the following equation $w_x = {E_x}\big/{q_x\left(1-q_x\right)}$ where E_x is the exposed-to-risk population at age x and q_x the theoretical probability of being married at age x. However the numerical illustrations of the weighted approach showed no significant improvement in the fit of the models, we thus adopted the non-weighted approach as easier. Hoem et al. (1981) also suggested a non-weighted approach for fitting the fertility rates of modern populations.

4. *Evaluation*

In order to evaluate the adequacy of the model proposed, we initially fit Model 1 to a variety of empirical data sets. For comparison reasons we also fit to the same data sets the Standard Coale-McNeil model, the Coale-McNeil one in its flexible form and the GLG model. In cases where the above models fail to adequately estimate the nuptiality pattern, we also fit the Mixture Coale-McNeil model and the Mixture Models 1 and 2.

All functions are fitted by means of a non-linear least-squares procedure and a Gauss-Newton optimization scheme. The Matlab built-in routine for non-linear parameter estimation "*lsqnonlin"* was used in order to find the unconstrained minimum of the unweighted sum of squares.

In the literature it has been a subject of long discussion about whether a period or a cohort approach should be adopted in the study of fertility and nuptiality. In addition the advantages and disadvantages of both approaches have been pointed out. It is accepted that period measures of nuptiality and fertility are subject to compositional and distributional 'distortions' such as those from flux in marital and parity composition and tempo effects (Kaneko, 2003). Although there have been made suggestions for correcting the distortions of the period measures (Bongaarts and Feeney, 1998; Kohler and Philipov, 2001; Kohler and Ortega, 2002; Ryder, 1964; 1980) cohort nuptiality measures which are free from these effects are of primary importance in demographic analysis. Nevertheless cohort measures have the disadvantage that they cannot be evaluated until the life course processes of the events are completed and therefore they do not provide information on the current situation of uncompleted phenomena. Therefore our applications are based on period data. This can be justified by several reasons. Firstly the period approach enables to examine and evaluate recent developments of demographic events as well as comparative analyses of these, while the cohort approach is by definition concerned with a longer-term development, as cohort trends and differences are accumulated during relatively long periods of time. Another basic reason for the adoption of period approach is that period data are readily available for analytical purposes, whereas cohort data frequently remain incomplete and difficult to reconstruct.

Therefore period single-year age-specific first-marriage rates for the female populations of Spain, Norway, Sweden, Finland, and Ireland for existing available years are used. These data were obtained from the Eurostat New Cronos database.

The results of our analyses concerning recent years are shown in the Appendix. The values of the estimated parameters of the evaluated models are given in Tables 1 to 4. The residual sums of squares are given in Table 5. Furthermore, the empirical and fitted age-specific first-marriage rates for selected years are depicted in Figures 1 to 6.

We initially compare the GLG model, the Coale-McNeil model in its flexible form (with 4 parameters), the standard Coale-McNeil model and the simpler alternative of our models (denoted as 'Model 1'). In the sequence we evaluate the mixture Coale-McNeil model as described in Liang (2000) as well as the mixture models proposed in this paper (denoted as 'Mixture Model 1' and 'Mixture Model 2').

Taking into account the results of our analyses we should mention that simple models generally provide poor fits at the tails and the peak of the age-specific first marriage rates. This fact has already been described by Liang (2000) who used Chinese data. However it has not been marked for modern European populations so far. Liang (2000) evaluated the double exponential distribution for Chinese women and found that this model failed in some cases to fit well the age-specific first marriage pattern of these populations. He found that in some cases the age peak value of the observed rates is not predicted well by the model. He explained that this is because of age heaping at first marriage, since Chinese people preferred to report their marriage as 18 years, before 1976. He also presented the case where the predicted distribution of the age at first marriage is almost a normal distribution but gave poor fits of the observed data. In this case the left-hand side of the curve of predicted data fitted well but it gave poor fits of the right-hand side of the curve. Liang (2000) made the plausible hypothesis that the observations may come from two different groups, a hypothesis that was initially developed by Chandola et al. (1999) for fertility data. According to this hypothesis one group has a double exponential distribution and the other has a different distribution. Therefore the observed data are the mixture of these two distributions. Finally, based on the results of Liang, in some populations the curve of first marriages is characterised by a bulge in the left side of the marriage distribution, because there is a group of women that marry earlier. Kaneko (2003) has also mentioned that these systematic deviations between observed and estimated rates are due to systematic error derived from simplification or insufficiency in model specification.

The observed and estimated rates of the GLG model, the Coale-McNeil model in its flexible form (with 4 parameters), the standard Coale-McNeil model and Model 1 are depicted in Figures 1 to 3 for the populations of Spain, Norway, Sweden, Finland and Ireland. Regarding our results similar conclusions with those of Liang can be drawn about the fit of simple models in European data. We observe that for all the populations examined, simple models fail to provide close fits of the observed rates over the whole age range. The misleading fits at the peak value of the marriage distribution might be related to the phenomenon of age heaping.

In the case of the above populations, simple models fail to closely estimate the tails and the peak value of the marriage distribution. From the graphical representation of the results, the existence of a bulge at young ages and another one at older ones are obvious in most recent Swedish data. The bulge at young ages appears around age 20, and the other one around age 40. We also observe that a bulge has started to appear for older people in the data sets of Finland and Norway and to a lesser extend for Ireland.

Some additional information for the quality of fits of simple models, is based on the values of the statistical criterion used, presented in Table 5. According to the values of this criterion, for the majority of the cases the Coale-McNeil model provides the best fits among the Standard Coale-McNeil model, the GLG model and Model 1. The second best fit is usually obtained by the GLG model that is mathematically equivalent to the Coale-McNeil model. According to this criterion, the Coale-McNeil model, and the GLG one provide somewhat closer fits to the data than Model 1.

Following the hypothesis of Liang (2000) that the observed data sets come from two or more different distributions, we then fit mixture models. A presentation of the results of mixture models is given in Figures 4 to 6. It is obvious that mixture models provide better fits that the simple models over the total age-span of the empirical data. We observe that Mixture Model 1 and Mixture Model 2 provide equivalent results in terms of fitting for the majority of the populations examined. In fact, according to the statistical criterion used, among the various alternatives Mixture Model 2 gives the best fits for all the cases except for Spain, 2002. Obviously, Mixture Model 1 and Mixture Model 2 provide better fits that the Mixture Coale-McNeil model, which underestimates the tails of the age-specific first-marriage distribution in many cases. It is also remarkable that all the mixture models failed to adequately estimate the marriage distribution for the Swedish female population for the year 2002. This might be related to the very distinct shape of the marriage curve, consisting of two bulges one at young ages and the other at a modest level so far, at older ages, and it might be an indication of great heterogeneity of women that marry. In fact it is an indication that this group consists from more than two subpopulations that have different characteristics regarding the age they marry.

From our analysis it comes out that simple models cannot closely fit first-marriage curve in modern European populations. As stated in the literature this might be an indication that the observed data come from two or more distributions indicating great heterogeneity in the population regarding its behaviour towards marriage. It is of course of interest to identify and describe the sources of heterogeneity in nuptiality. In order to explain this heterogeneity, different hypotheses in each population separately should be tested based on the country's socio-political and historical background. Regarding Spain, a hypothesis that could explain the existence of heterogeneity in first-marriages might be associated to some extent to the growth of immigrant populations from third-world origins with higher and earlier patterns of first-marriages compared with those of the indigenous populations. Religiosity may also contribute to the existence of this bulge in the age-specific first-marriage distributions. Also in the Irish population, this may be related to the fact that Catholicism is very powerful in the Irish population. In Scandinavian countries, heterogeneity in the first-marriage pattern could also be associated to the second-generation immigrants from Arabic and Eastern countries. Finally education might play an important role in the existence of the bulge at earlier ages of first-marriage curve, indicating that it consists of two different groups.

As mentioned above a variety of factors related to the socioeconomic and cultural background of male and female populations may contribute to the appearance of the heterogeneity in the first-marriage curve. However in order to be able to verify or reject all these hypotheses about heterogeneity in the first-marriage curve further research based on empirical evidence, is required.

5. *Conclusions*

Various models have been proposed in the literature for estimating the first-marriage age distribution. Recent evidence indicates that these models are inadequate for estimating the pattern of first marriages in many modern populations. A probable hypothesis for explaining that, is that these populations are heterogeneous regarding their behaviour towards the age at marriage. In such populations a new first-marriage pattern has started to arise, which consists of a second bulge at earlier ages. So far, this fact has been described for Chinese women by Liang (2000) and a mixture model was proposed for dealing with this problem.

In this work, three versions of a parametric model for describing the age-specific nuptiality pattern are proposed. Initially, a simple parametric formula for fitting the age-specific first-marriage distribution, denoted as Model 1 is proposed. Then two mixture versions of this formula denoted as Mixture Model 1 and Mixture model 2 are proposed for capturing the distorted pattern of modern populations. In order to evaluate the adequacy of the models proposed, we fit the three alternative formulae to a variety of data sets of several populations. Furthermore we compare these to existing models in the literature such as the Standard Coale-McNeil, and the Coale-McNeil model in its flexible version, as well as the Generalized Log Gamma model and the Mixture Coale-McNeil model.

A general finding of our evaluation is that simpler models cannot provide close fits of the tails of the curve and in many cases they also fail to capture the peak value of the marriage distribution. Based on the values of the statistical criterion, among the simple models, the Coale-McNeil and the GLG models provide the best fits while the Standard Coale-McNeil model seems quite insufficient for describing such data. Therefore the use of mixture models is required.

Regarding mixture models, both Mixture model 1 and Mixture model 2 provide successful fits of the age-specific first-marriage distributions while the Mixture Coale-McNeil model gave poor fitting results in most cases. It is also remarkable that although Mixture model 1 requires a less parameter to be estimated than the Mixture Coale-McNeil model it provides better results than the later. Furthermore Mixture model 2 and the Mixture Coale-McNeil model require the same number of parameters, but the former is much simpler and easier to interpret. In addition the parameters of the Mixture model 1 and Mixture model 2 have an explicit demographic interpretation while they can be utilized for understanding the shape of the first-marriage pattern and for comparisons through time and place, between populations or population subgroups.

An interesting finding of our analysis is that for Swedish populations except for the bulge that appears at young ages a second one also appears at a modest level at the old ages, indicating stronger heterogeneity. This heterogeneity also characterises the marriage distributions of other female populations such as Finland and Norway, indicating that in these populations there is a group of women that is getting married at a relatively old age i.e. around the 40s which to some extent may be related to the modernisation of society regarding women and their role in the society, as well as the behaviour in regards of their professional career. Another possible explanation that needs further research is that these populations have been heterogeneous due to the growth of second-generation immigrant populations from third-world origins with higher and earlier patterns of first marriages than the indigenous populations. Other factors associated with this heterogeneity might be differences in educational level and economic status, and religion of various groups in these populations.

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APPENDIX A

Table 1: Estimated parameters of the Coale-McNeil model and the GLG model for the female populations

	Coale- McNeil model					GLG model		
	α	c	λ	μ	a	b	l	u
Spain 2002	0,95	0,08	0,54	58,77	0,01	3,67	-0,48	26,01
Norway 2002	0,36	0,14	0,59	32,52	0,61	4,39	-0,71	26,12
Sweden 1997	0,28	0,17	0,41	30,34	0,47	4,58	-0,75	27,21
2002	0,32	0,15	0,39	32,81	0,39	4,57	-0,71	27,44
Finland 1998	0,34	0,16	0,56	30,35	0,61	4,26	-0,73	25,56
Ireland 1998	0,41	0,23	0,61	28,43	0,71	3,21	-0,75	26,08

Table 2: Estimated parameters of the Standard Coale-McNeil model and Model 1

	Standard Coale - McNeil			Model 1			
	u	b	c	c_1	μ	σ_{11}	σ_{12}
Spain 2002	19,51	0,57	0,75	0,06	26,09	4,83	5,66
Norway 2002	18,28	0,62	0,93	0,05	25,83	5,03	0,05
Sweden 1997	19,15	0,42	0,96	0,03	27,01	5,34	8,04
2002	19,48	0,41	0,95	0,03	27,11	5,19	8,12
Finland 1998	17,96	0,58	0,90	0,05	25,23	4,80	7,64
Ireland 1998	20,43	0,62	0,67	0,07	25,72	3,49	5,96

Table 3: Estimated parameters of the Mixture Coale-McNeil model

	Mixture Coale-cNeil model						
	α_1	λ_1	m	μ_1	λ_2	α_2	μ_2
Spain 2002	0,05	0,03	0,5	12,88	0,11	0,81	44,44
Sweden 1997	0,11	0,03	0,6	21,69	0,21	0,24	27,87
2002	0,19	0,24	0,5	26,92	0,41	0,003	61,07
Finland 1998	0,004	0,07	0,5	14,46	0,15	0,39	32,21
Ireland 1998	0,027	0,01	0,4	9,763	0,23	0,47	29,41

Table 4:Estimated parameters of Mixture model 1 and Mixture model 2

	Mixture Model 1						Mixture Model 2						
	c_1	m_1	σ_{11}	c_2	m_2	σ_2	c_1	m_1	σ_{11}	σ_{12}	c_2	m_2	σ_2
Spain													
2002	0.04	26.18	3.99	0.02	27.38	8.69	0.04	26.19	4.02	3.98	0.02	27.40	8.69
Sweden													
1997	0.02	27.27	4.79	0.01	30.79	10.40	0.03	27.51	5.38	4.29	0.01	31.36	9.60
2002	0.02	27.41	4.55	0.01	30.58	10.24	0.02	27.32	4.37	4.73	0.01	30.46	10.45
Finland													
1998	0.05	25.57	5.14	0.01	31.95	8.48	0.05	25.49	5.04	5.49	0.01	32.62	8.78
Ireland													
1998	0.06	26.02	3.73	0.02	29.76	6.73	0.06	25.85	3.37	4.42	0.01	29.71	8.18

Table 5: Sum of Squared Errors

SSE*10^8	Model	Standard Coale-McNeil model	GLG model	Coale-McNeil model	Mixture Coale-McNeil model	Mixture Model 1	Mixture Model 2
Spain							
2002	14376	46959	14321	12446	6250.4	917.6	934.5
Norway							
1998	11477	25160	8660	8350	10731	5110	5060
Sweden							
1997	11301	16568	8872.9	9152.2	8481.6	2688.9	2428.7
2002	20068	32185	18168	17859	26983	3291.1	2891
Finland							
1998	13410	23595	8527	8435.5	6049.4	3841.7	3704.7
Ireland							
1998	18144	62918	19153	17110	8562.6	1765.5	1682.1

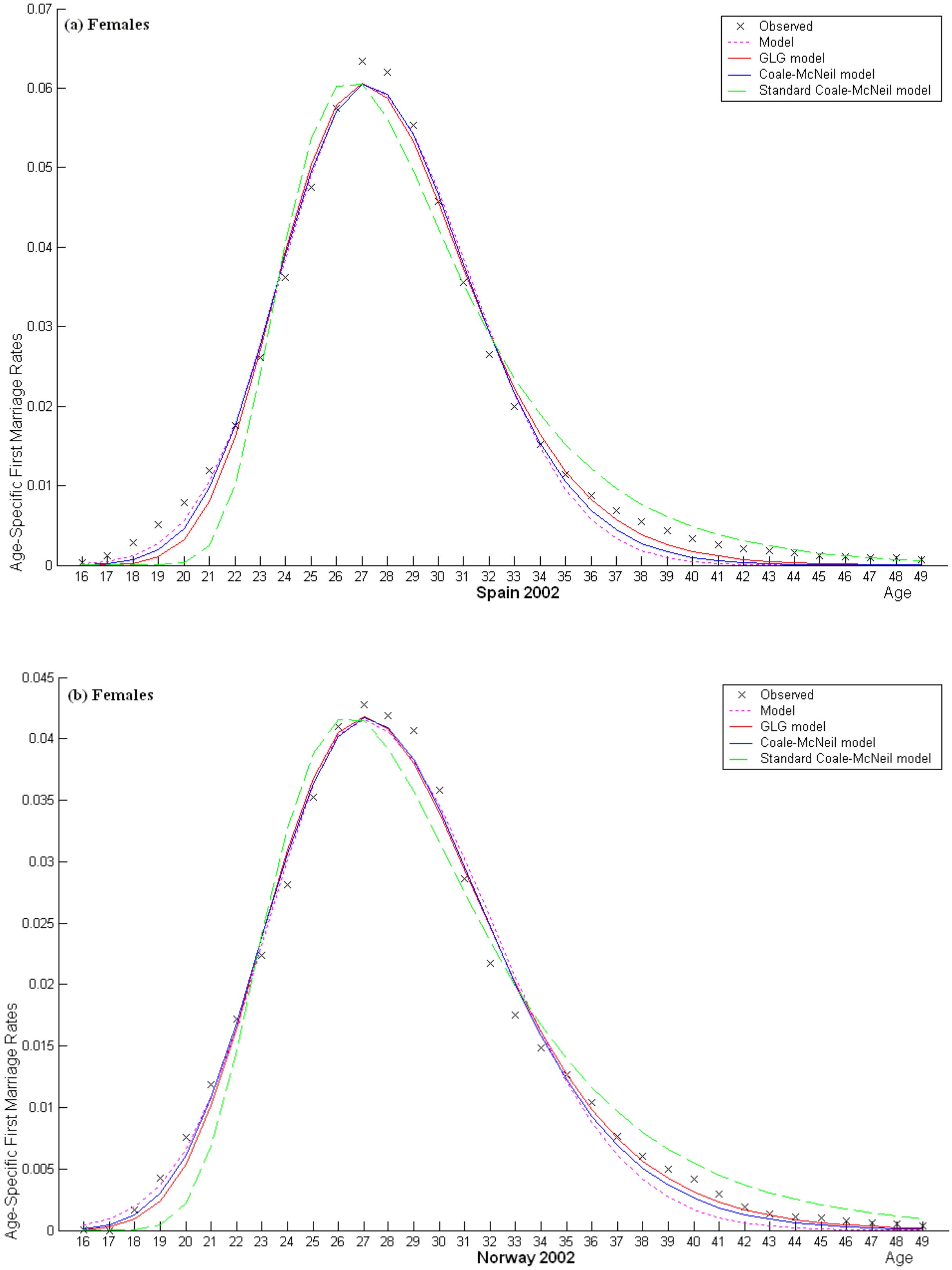


Figure 1: Observed and estimated age-specific first marriage rates of females, using simple models for a) Spain, 2002 and b) Norway, 2002.

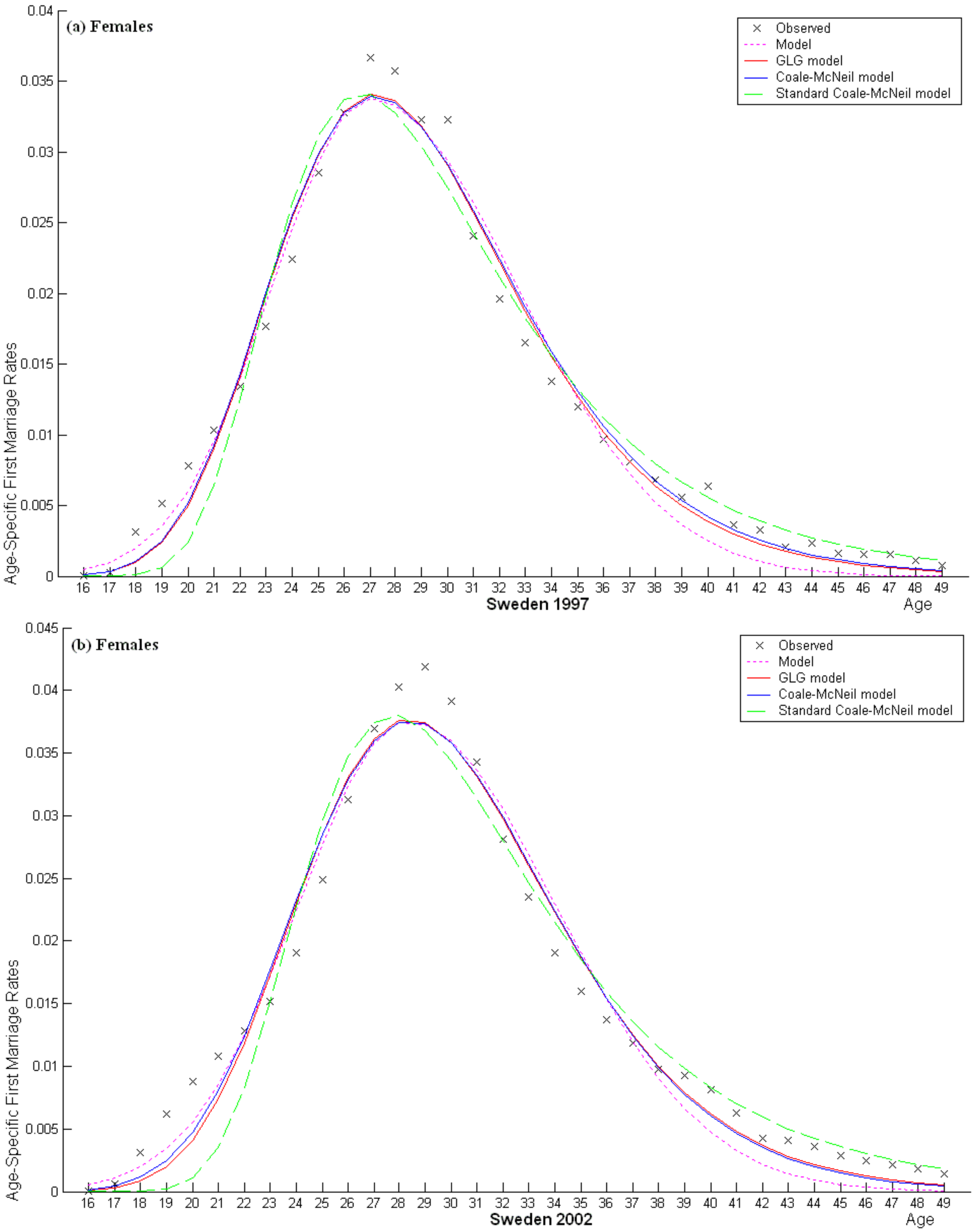


Figure 2: Observed and estimated age-specific first marriage rates of females, using simple models for a) Sweden, 1997 and b) Sweden, 2002.

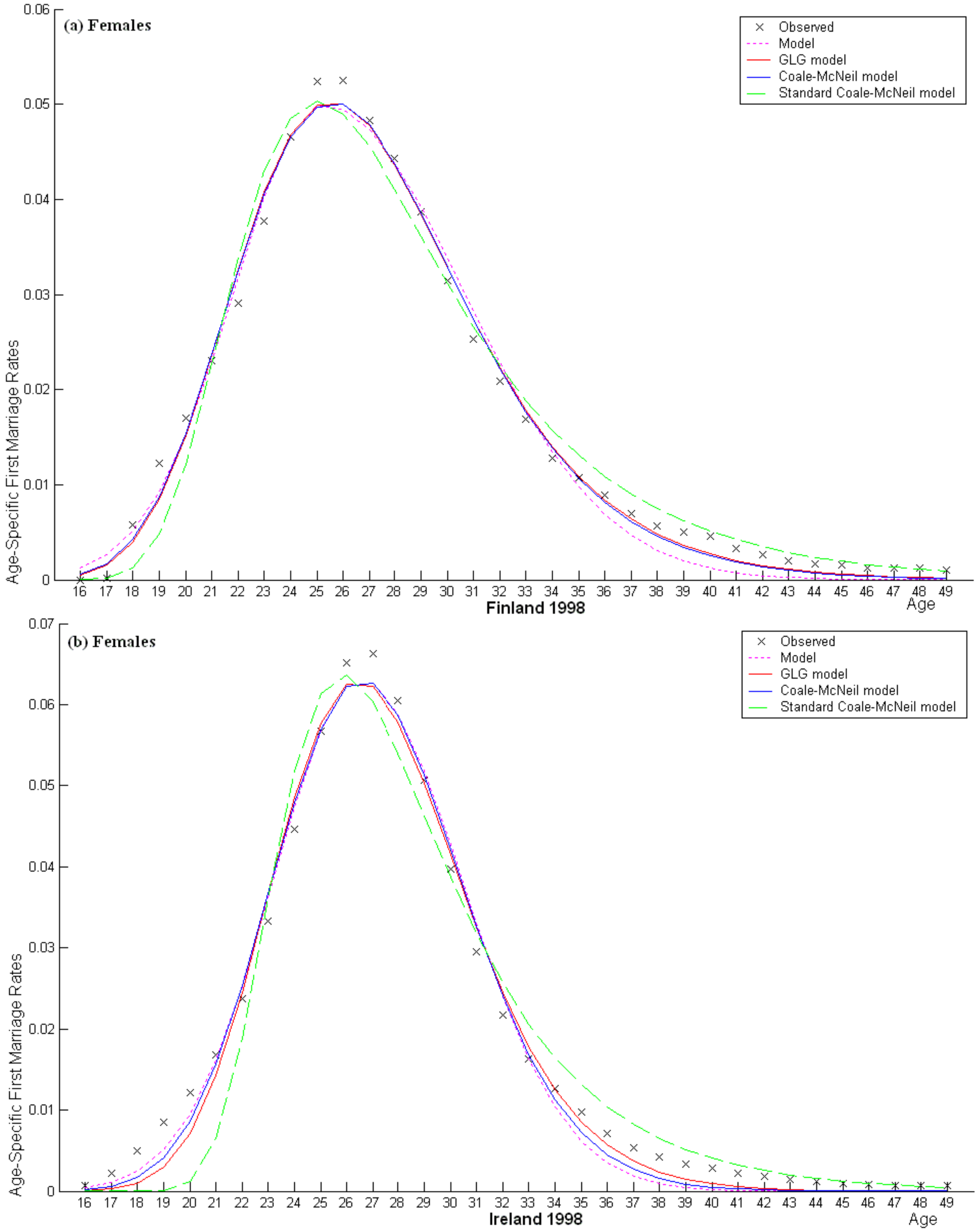


Figure 3: Observed and estimated age-specific first marriage rates of females, using simple models for a) Finland, 1998 and b) Ireland, 1998.

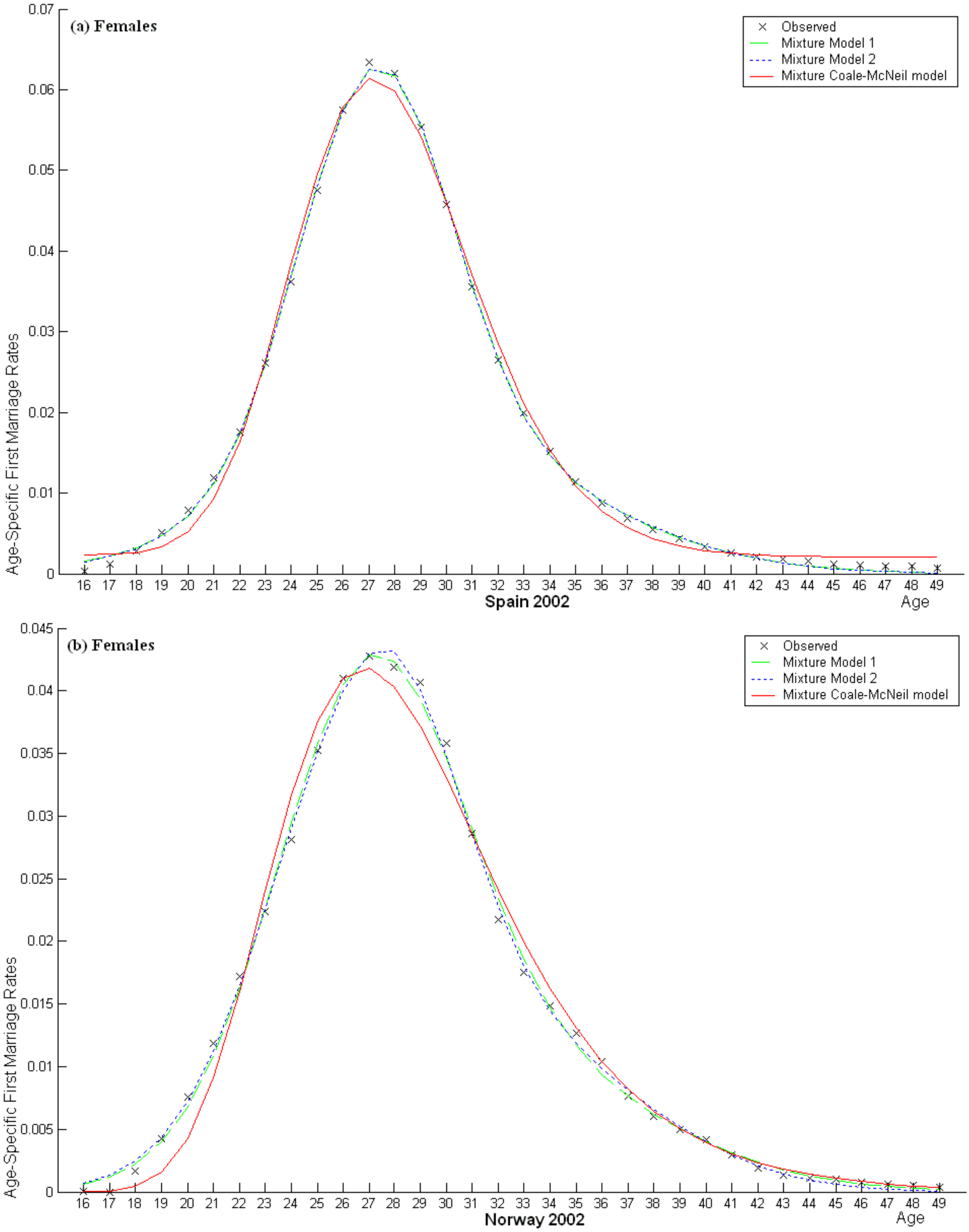


Figure 4: Observed and estimated age-specific first marriage rates of females, using mixture models for a) Spain, 2002 and b) Norway, 2002.

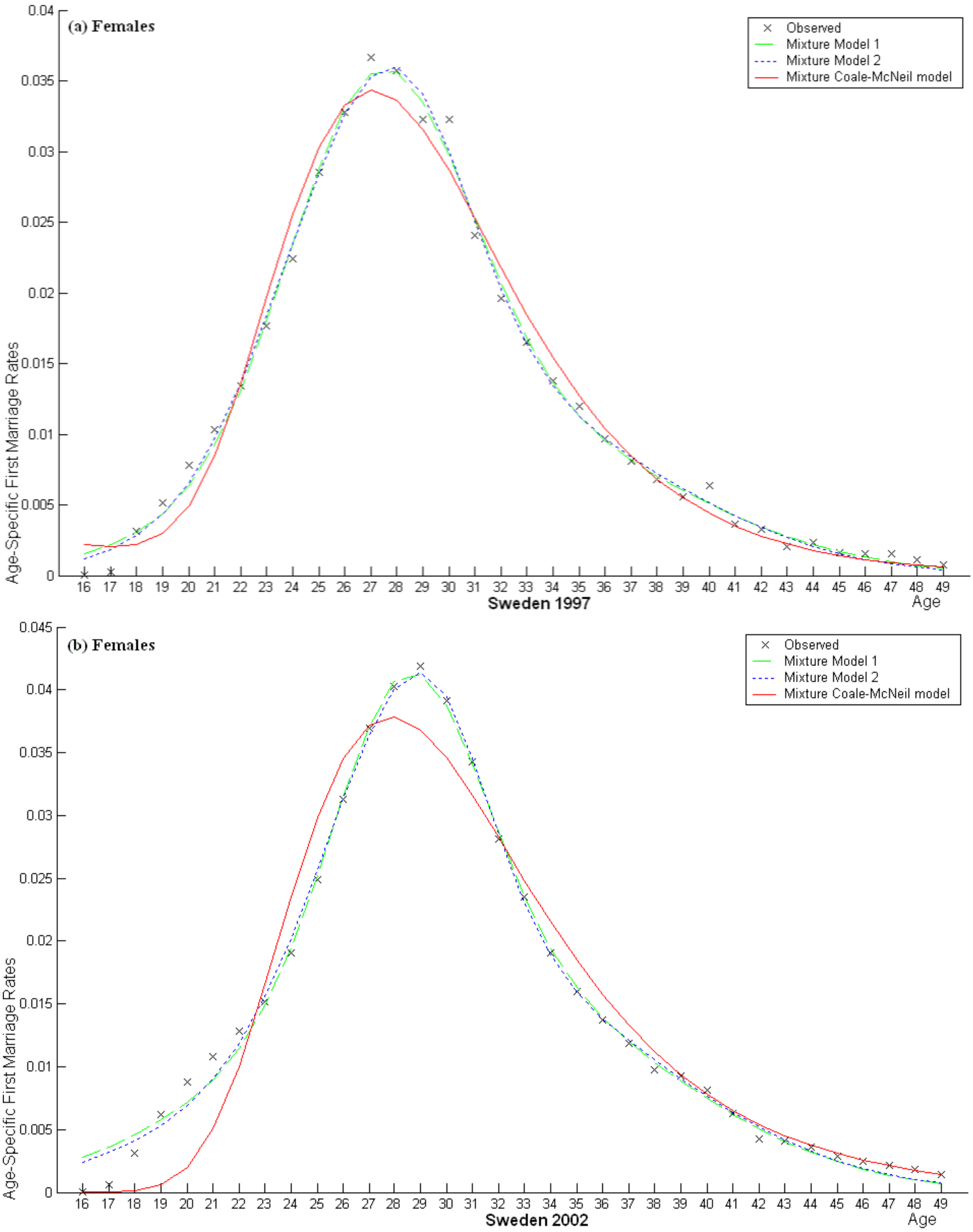
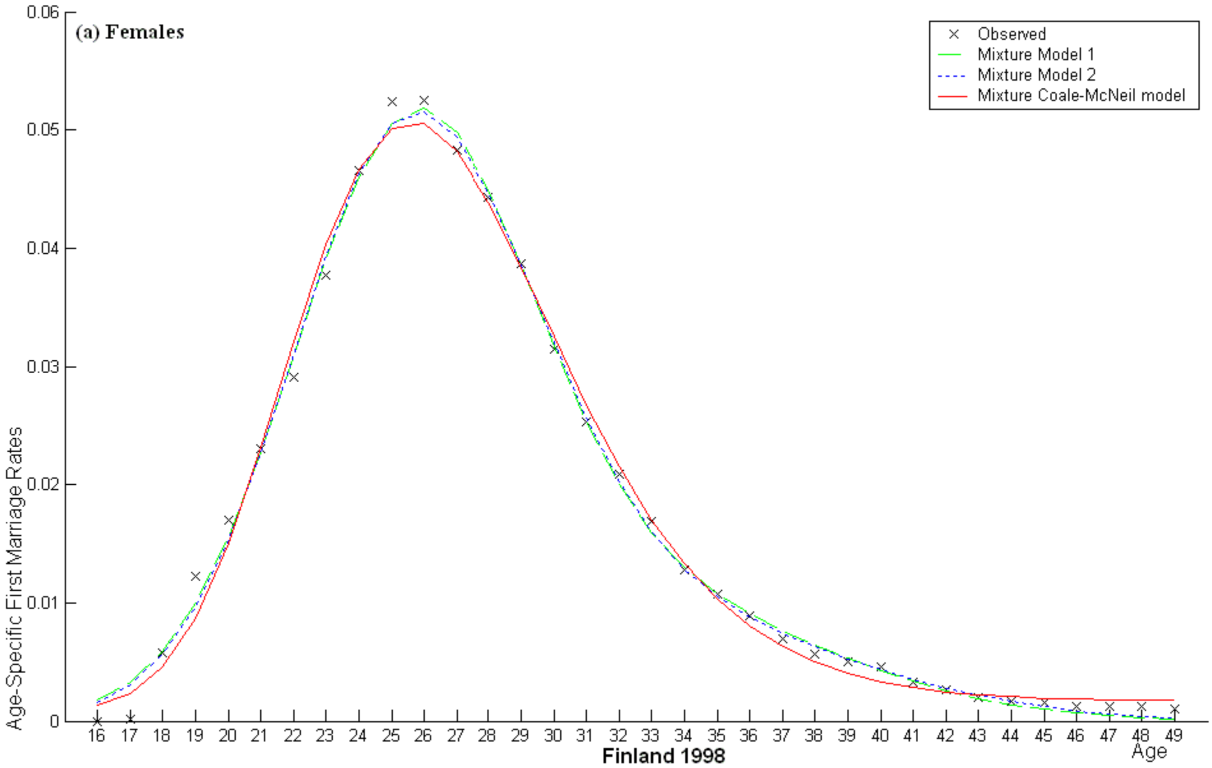


Figure 5: Observed and estimated age-specific first marriage rates of females, using mixture models for a) Sweden, 1997 and b) Sweden, 2002.



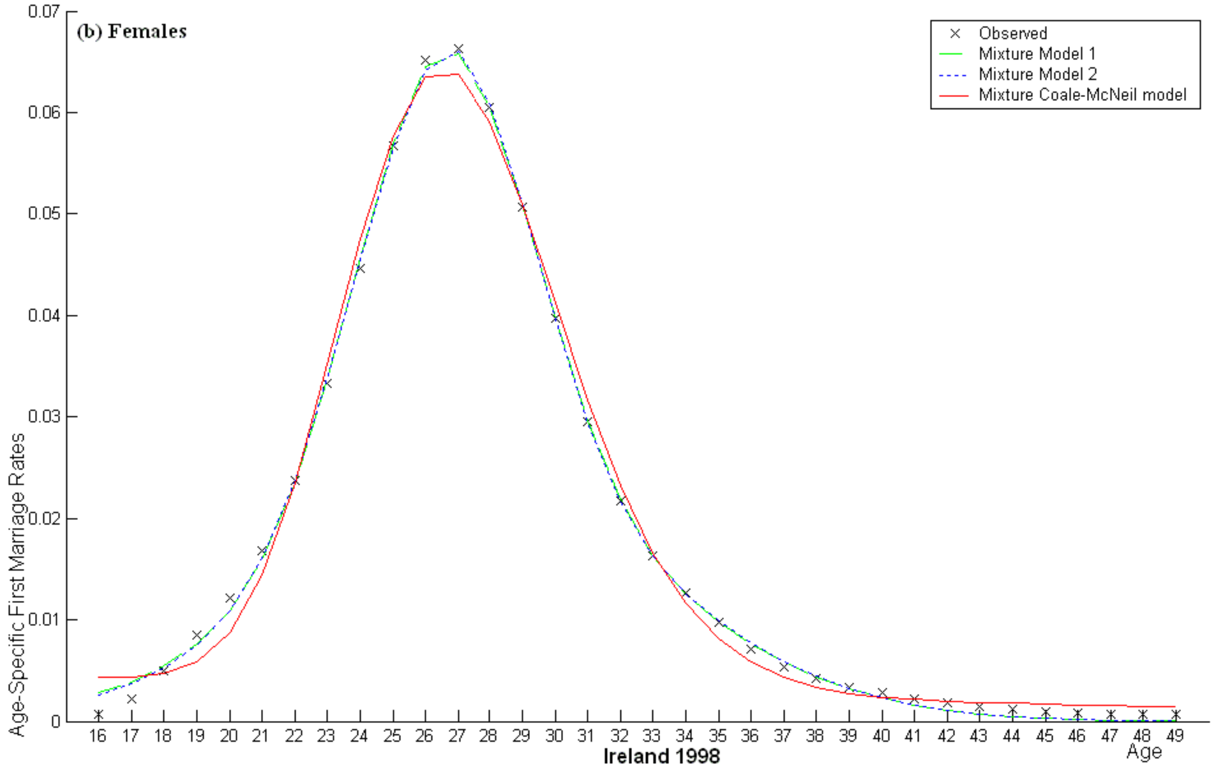


Figure 6: Observed and estimated age-specific first marriage rates of females, using mixture models for a) Finland, 1998 and b) Ireland, 1998.