

# The narrowing sex differential in life expectancy: mortality improvement and its efficiency

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**Introduction.** Over the past century the sex differential in life expectancy ( $e^o$ ) has widened in favor of women. A reversal of this differential has been found in some developed countries since the early 1970s (Trovato and Lalu, 1996; Trovato and Heyen, 2005). Enormous attempts have been made to explain the narrowing sex differential in  $e^o$  by investigating causes of death and behavioral and medical factors that are related to sex differences in mortality and  $e^o$  (e.g., Waldron, 1986; Trovato and Lallu, 1998; Trovato and Heyen, 2005; Preston and Wang, 2006). Unlike those studies, Gleit and Horiuchi (2007) carried out a demographic analysis to reveal the mechanism underlying the reversal. Their argument is that the narrowing of sex differential is due primarily to the sex difference in the age pattern of mortality rather than declining sex ratios in mortality. In the present study, we did a similar decomposition analysis, which was somewhat different from Gleit and Horiuchi's, to examine the driving forces underlying the narrowing sex differential in  $e^o$ .

Based on a decomposition of change over time in life expectancy by Vaupel and Canudas-Romo (2003), we applied Kitagawa's method (1955) to decompose the change in sex differential in  $e^o$  into two components, one of which captures the sex difference in mortality improvement and the other reflects the sex difference in efficiency of mortality improvement. The results indicate that (1) the changing sex differential has been highly correlated to sex difference in mortality change; (2) in narrowing sex differential in  $e^o$ , mortality reductions have performed much more actively than the sex difference in efficiency of mortality; (3) the role by sex difference in mortality change will continue to be dominant in the future, particularly considering that male  $e^o$  has been getting close to female one.

**Method.** The sex differential in  $e^o$  at time  $t$  is given by

$$\Delta e^o(t) = e_f^o(t) - e_m^o(t), \quad (1)$$

where  $e_f^o$  and  $e_m^o$  stand respectively for female and male life expectancies, while  $\Delta e^o$  is the sex differential in  $e^o$ . Then the change over time in the sex differential can be written as

$$\dot{\Delta e^o}(t) = d\Delta e^o(t)/dt = de_f^o(t)/dt - de_m^o(t)/dt = \dot{e}_f^o(t) - \dot{e}_m^o(t). \quad (2)$$

If  $\dot{\Delta e^o}(t)$  is positive, then the sex differential widens, and the sex differential narrows if  $\dot{\Delta e^o}(t)$  is negative. The above equation also means that the change in sex differential in  $e^o$  is equivalent to sex difference in change in life expectancy.

Recall a decomposition of change in life expectancy over time by Vaupel and Canudas-Romo (2003)

$$\dot{e}^o(t) = de^o(t)/dt = \int_0^\omega \rho(x,t)e(x,t)f(x,t)dx, \quad (3)$$

where  $\rho(x,t) = -\frac{\partial \ln \mu(x,t)}{\partial t}$  is the rate of reducing mortality,  $e(x,t) = \frac{\int_x^\omega \ell(a,t)da}{\ell(x,t)}$  the remaining life expectancy at age  $x$ ,  $f(x,t) = \ell(x,t)\mu(x,t)$  the death distribution of life table population,  $\mu(x,t)$  the mortality at age  $x$ , and  $\ell(x,t) = \exp(-\int_0^x \mu(a,t)da)$  the survival function at age  $a$ . Oeppen, 2008 proposed a concept of efficiency of the age pattern of mortality change. Let

$$\varepsilon(x,t) = e(x,t)f(x,t). \quad (4)$$

Then, an ‘‘efficient’’ age pattern of mortality reduction should be the one that produces greater mortality reductions at ages where  $\varepsilon(x,t)$  fractions are greater. It follows from (3) and (4) that

$$\dot{e}^o(t) = \int_0^\omega \varepsilon(x,t)\rho(x,t)dx. \quad (5)$$

Substituting (5) into (2) yields

$$\dot{\Delta e^o}(t) = \int_0^\omega \varepsilon_f(x,t)\rho_f(x,t)dx - \int_0^\omega \varepsilon_m(x,t)\rho_m(x,t)dx$$

where the subscripts  $f$  and  $m$  represent female and male respectively.

Following from Kitagawa’s method, (6) can be rewritten as

$$\begin{aligned} \dot{\Delta e^o}(t) &= \int_0^\omega \left( \frac{\varepsilon_f(x,t) + \varepsilon_m(x,t)}{2} \right) (\rho_f(x,t) - \rho_m(x,t))dx + \\ &\quad \int_0^\omega \left( \frac{\rho_f(x,t) + \rho_m(x,t)}{2} \right) (\varepsilon_f(x,t) - \varepsilon_m(x,t))dx \\ &= \Delta(\rho) + \Delta(\varepsilon), \end{aligned} \quad (6)$$

where the first term,  $\Delta(\rho)$ , is the product of the sex difference in mortality change and the average of efficiencies for both sexes. This term captures the effect of mortality change on sex differential in  $e^o$ . Note that a positive  $\Delta(\rho)$  simply means that mortality improvement benefits more to women than to men, and the sex difference in mortality decline will *widen* sex differential in  $e^o$ ; and a negative  $\Delta(\rho)$  will do the reverse.

The second term,  $\Delta(\varepsilon)$ , reflects the impact of sex gap in efficiency of mortality improvement

<sup>1</sup>. A negative  $\Delta(\varepsilon)$  means that the relatively high efficiency of mortality improvement among men will narrow the sex difference in  $e^o$ .

**Results.** The decomposition as given in (6) is applied to selected countries from Human Mortality Database (2008). The results indicate that the sex gap in mortality improvement is highly correlated with the change in sex differential in  $e^o$  (Figure 1). Compared to the sex difference in efficiency, mortality changes play the dominant role of changing, either widening or narrowing, sex differential in  $e^o$ .

The Figure 2 depicts the change over time in two components of decomposition as in (6),  $\Delta(\rho)$  and  $\Delta(\varepsilon)$ . First, starting from around the 1950s, the sex difference in mortality decline,  $\Delta(\rho)$ , has followed the trend of decline, though with a little fluctuation in some periods. Particularly in the early 1970s or the 1980s,  $\Delta(\rho)$  even became negative in some countries like England and Wales, Canada, the Netherlands, USA, etc. Note that the negative  $\Delta(\rho)$  simply means that mortality improvement benefited more to men than to women. Therefore, the sex gap in mortality improvement narrowed the sex differential in  $e^o$  in those countries.

Second, the sex difference in efficiency of mortality change,  $\Delta(\varepsilon)$ , varied modestly over the past fifty years, particularly compared to  $\Delta(\rho)$ . This further echos the above finding from Figure 1 that the change in sex differential in  $e^o$  is highly associated with the sex difference in mortality decline rather than that in efficiency. Moreover,  $\Delta(\varepsilon)$  is mostly negative, suggesting that men usually had higher efficiency of mortality improvement than women. This means that, even with the same progress of mortality decline, men would catch up with women in life expectancy because of their relatively high efficiency. In this sense, mortality decline in favor of men can just accelerate the narrowing of sex differential in  $e^o$ .

As  $\Delta(\rho)$  declines over time,  $\Delta(\varepsilon)$  will make relatively big contribution to the narrowing of sex differential in  $e^o$ . In some cases where the absolute value of  $\Delta(\varepsilon)$  is greater than  $\Delta(\rho)$ , sex differential in  $e^o$  can primarily be attributed to  $\Delta(\varepsilon)$ . This is what occurred in England and Wales in the 1970s, USA in the late 1970s, Canada in the 1980s, and the like.

Although  $\Delta(\varepsilon)$  outweighed the  $\Delta(\rho)$  in narrowing sex differential in  $e^o$  in some periods, we have to realize that the important factor is the sex difference in mortality decline. First, according to the history of human mortality so far, the sex difference in efficiency has changed very modestly, especially compared to that of mortality decline. Note that the variation of  $\Delta(\rho)$  is much bigger than that of  $\Delta(\varepsilon)$ . Second, as male life expectancy is getting close to female one, the sex difference in efficiency will decrease accordingly. This is determined by the efficiency function,  $\varepsilon(x) = e(x)f(x)$ .

Considering that the sex differential in  $e^o$  hovers around from four to seven years currently, there is still spaces to further decrease. If we further ask what kind of forces, mortality improvement or efficiency of mortality change, may primarily push the progress of narrowing sex differential in  $e^o$ . The answer should be the mortality improvement in favor of men, as suggested by the results of the present study.

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<sup>1</sup>In Gleijer and Horiuchi (2007) this was called as the effect of age pattern of mortality changes. Despite the difference in interpretation,  $\varepsilon(x)$  acts as an efficiency function in effect, indicating how much life expectancy would be increased given a proportionate decline in mortality at age  $x$ .

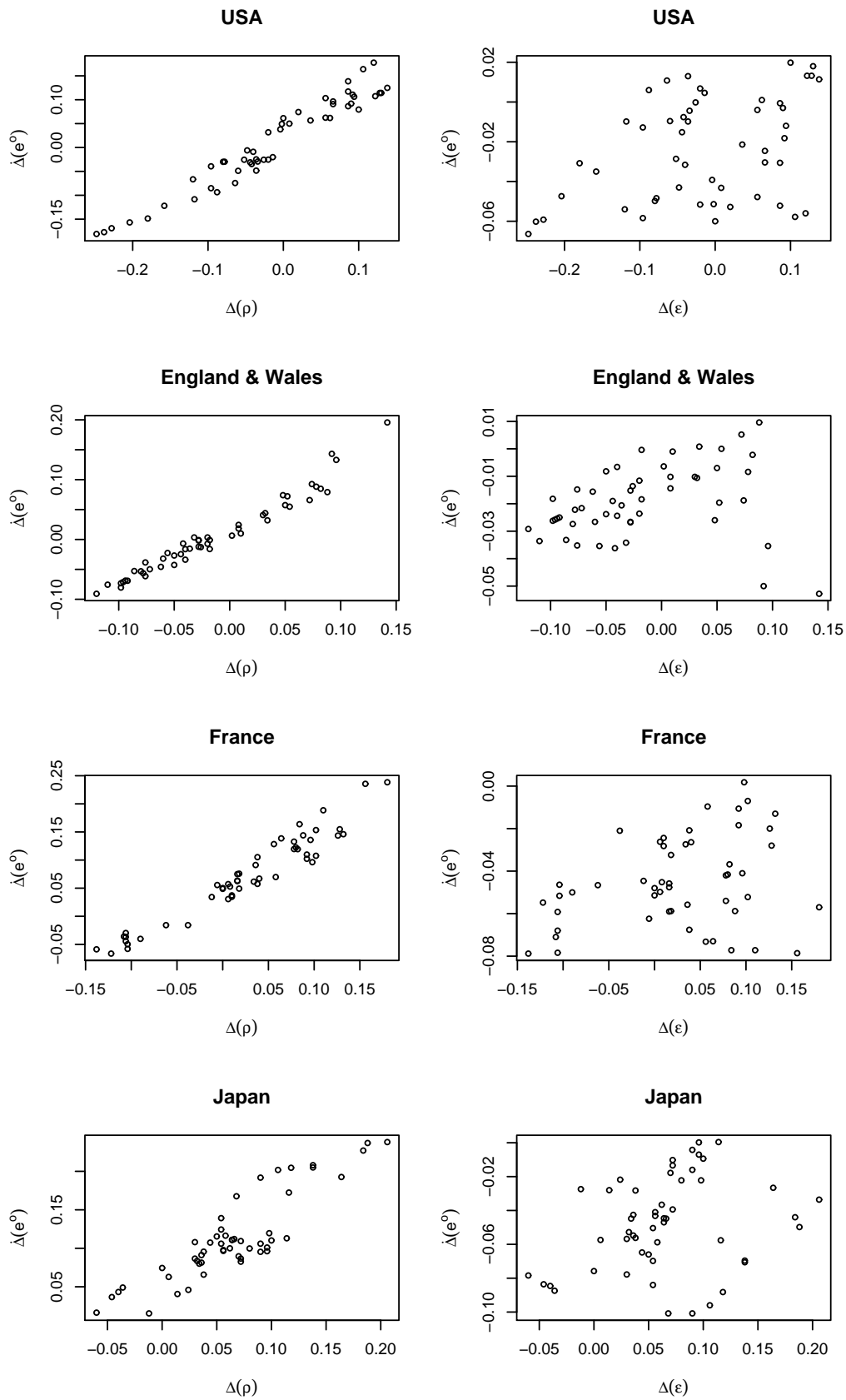


Figure 1: Correlation between change in sex differential,  $\Delta e^o$  and two components of decomposition,  $\Delta(\rho)$  and  $\Delta(\epsilon)$ .

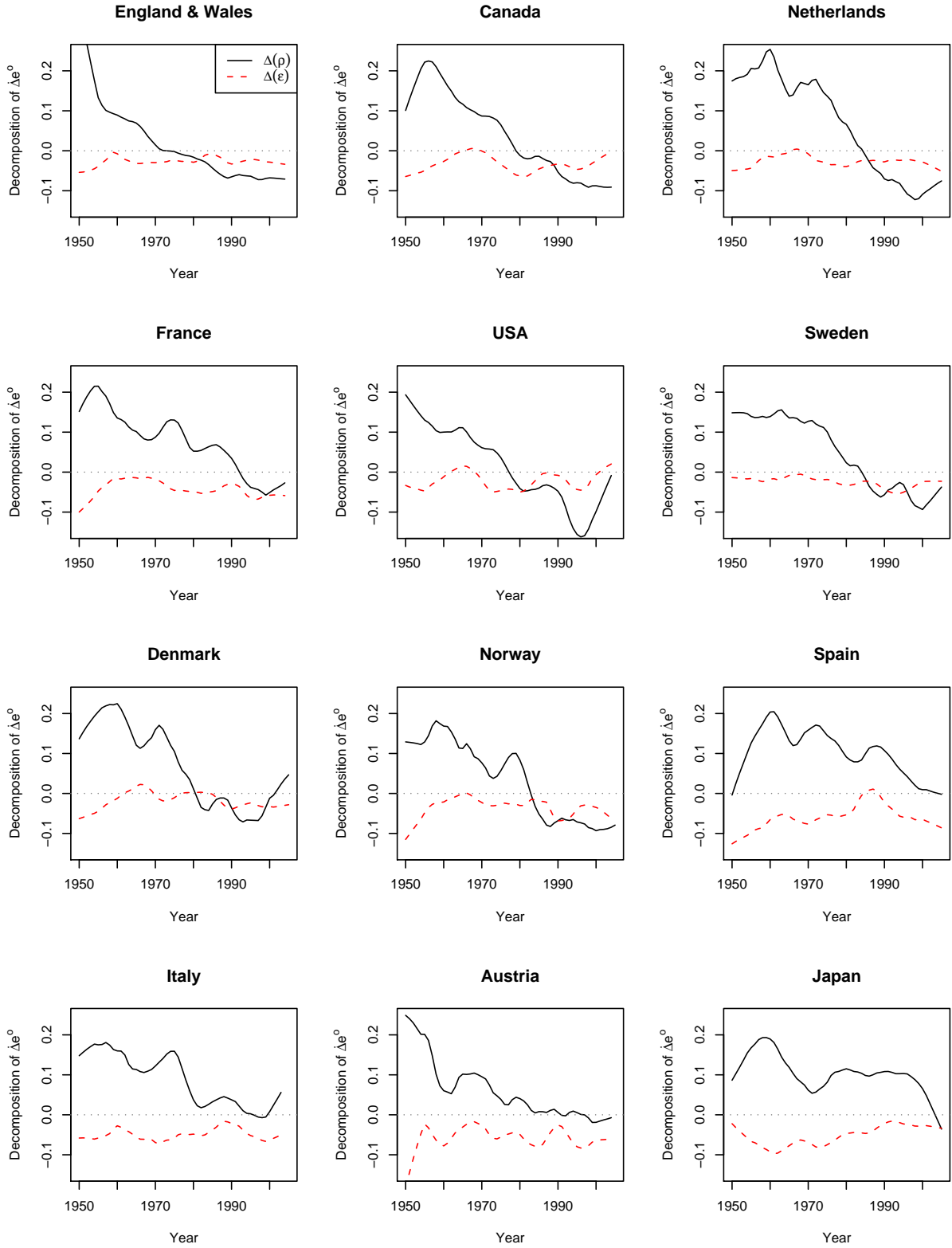


Figure 2: Decomposition of change in sex differential in  $e^o$ .

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