New Ways to Look at Mortality Deceleration

Extended Abstract

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Abstract

Throughout most of adult lifespan, mortality increases linearly on a logarithmic scale. At advanced ages, though, mortality levels off resulting in overestimating mortality if a Gompertzian shape is assumed. Most commonly this mortality deceleration is measured by the life-table aging rate, introduced by Horiuchi and colleagues, but also other methods were used previously. In this paper we present two alternative approaches to determine the age when mortality deceleration starts: 1) the age when mortality acceleration is at its maximum and b) the age when observed mortality deviates significantly from the exponential increase. After a theoretical justification, we show with empirical data that these two new methods are not only intuitively but also practically appealing.

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Background and Aim of Our Research

Throughout most of adult lifespan, mortality increases exponentially and is well approximated by Gompertz' equation $\mu(x) = \alpha e^{\beta x}$, where x indicates age. Gompertz (1825, p. 517) attributed this age-specific exponential increase to "a deterioration, or an increased inability to withstand destruction" (see also Vallin and Berlinguer (2006, p. 96)). In 1960, Strehler and Mildvan showed in a more formal approach that the age-related mortality increase, as measured in the β parameter, can be interpreted in terms of a loss of vitality and environmental stress.

Eventually, the exponential increase levels off at higher ages as shown in Figure 1 (page 12). As a result at these age, the Gompertz model, as indicated by the solid lines, is overestimating mortality for both females and males. Models with an implicit logistic structure reflect the observed mortality much better (Bongaarts, 2005; Thatcher, 1999; Thatcher et al., 1998; Zeng Yi and Vaupel, 2003).

What is the reason for the levelling off? Data problems, e.g. an underreporting of deaths, may occur as shown, for example by Elo and Preston (1997). It can not be considered a general explanation, though, since the same pattern of deviation from the log-linear trend has been observed in countries with highly reliable data and in non-human population in controlled environments (e.g. Vaupel and Carey, 1993; Vaupel et al., 1998).

If data problems can be excluded there are basically two explanatory frameworks (see, for instance, Horiuchi and Wilmoth, 1998). The *individual risk hypothesis* argues that the rate of aging on the individual level slows down at advanced ages. Alternatively, the *heterogenity hypothesis* emphasizes the compositional change of a population with increasing age rather than a change on the individual level. Due to higher mortality of frailer individuals, the population becomes more selected with increasing age, resulting in death rates increasing more slowly with age than in homogenous populations (Vaupel et al., 1998, 1979; Vaupel and Yashin, 1985).

The goal of our paper is not to test which one of the two theories is supported by empirical data. Our aim is rather to present previous approaches to measure deceleration in the age-related mortality increase and suggest alternative methods. After their theoretical justification for using those methods, we apply all methods in a case study. These empirical results show that the new methods are not only theoretically appealing but mark also prominent positions on the age-specfic curve of the force of mortality in a real case.

Measuring Mortality Deceleration

Visual Measurement

This approach refers to the "common exercise in mortality research to calculate the logarithms of age-specific death rates and plot them against age" (Horiuchi and Coale, 1990, p. 246).[1] The problem is, however, that it is rather difficult to tell from visual inspection alone exactly the onset of the age-related deceleration.

Relative Derivative

Instead, Horiuchi and Coale (1990) suggest to use the life-table aging rate $k(x)$ which is the relative derivative of the force of mortality $\mu(x)$:

$$k(x) = \frac{d \log\left(\mu(x)\right)}{dx} \tag{1}$$

Using the life-table aging rate (LAR) appears to be the most commonly used approach to measure the deceleration in the age-related mortality increase (Carey and Liedo, 1995; Horiuchi and Coale, 1990; Horiuchi and Wilmoth, 1997, 1998; Kannisto, 1996).

Despite its widespread adoption, we think that the relative derivative of the force of mortality,[2] is not the only desirable way to measure mortality deceleration at advanced ages.

Absolute Derivatives

Speed Let's assume that the force of mortality by age is the (one-dimensional) position of an object over time. The first derivative of the object's position with respect to time is speed (Feynman et al., 1963). One way to measure the onset of deceleration of the age-related mortality increase is, hence, to estimate the age when "mortality speed" is at its maximum (i.e. the age when the second derivative is zero and the third derivative is negative at this age). This is not a new suggestion of us. Despite assuming a different shape of the force of mortality and pursuing a different estimation strategy, Lynch and Brown (2001) used this idea implicitly in their paper on mortality compression and deceleration.

[1] To avoid confusion it should be pointed out that Horiuchi and Coale (1990) do not propose this method.

[2] Which is, of course, the derivative of the log hazard as denoted in Equation 1.

Acceleration The first new method which we suggest is to go one step further than to look at the speed of the force of mortality: The age when mortality acceleration reaches its maximum can be considered the onset of mortality deceleration. As we know from physics, acceleration is the second derivative, and the point of maximum acceleration is the age when the third derivative is zero and the fourth derivative is negative at this age. To our knowledge, maximum mortality acceleration has not been used previously to measure mortality deceleration.

Deviation from the Gompertz-Makeham Curve

Our second and last suggestion is unrelated to derivatives. Rather, from an intuitive perspective, we argue that mortality starts decelerating when actual mortality falls significantly below a Gompertz-Makeham curve. This is most easily done by a) assuming an alternative parametric model which fits mortality better at advanced ages (e.g. a logistic shape), b) fitting a corresponding model and c) looking at the age when the upper band of the confidence band (chosen at an arbitrary level; for example 99%) of the alternative model does not anymore include the Gompertz-Makeham fit. It should be noted that the choice of alternative mortality model is crucial in this case: depending on the model, the age when mortality deviates significantly from the Gompertz-Makeham curve can vary greatly.

Empirical Case Study

Data and Methods

We use mortality data for women in England & Wales for the year 2004. The data have been downloaded from the Human Mortality Database (University of California, Berkeley (USA), and Max Planck Institute for Demographic Research, Rostock, (Germany), 2008). We assume that the force of mortality by age follows a logistic shape:

$$\mu(x) = \frac{\alpha e^{\beta x}}{1 + \alpha e^{\beta x}} + \gamma \qquad (2)$$

This model is appealing not only because it fits mortality at adult and old ages remarkably well with only three parameters (Bongaarts, 2005; Thatcher, 1999). Moreover, as shown first (to our knowledge) by Beard (1971),[3] Equation 2 expresses the observed mortality of a popu-

[3] Beard refers in this publication from 1971 to earlier work of himself where he has shown it.

lation where each individual experiences a Gompertz-Makeham hazard with a Gamma frailty distribution for the individuals.

The actual estimation follows the standard maximum likelihood approach as outlined, for example, in Zeng Yi and Vaupel (2003), Thatcher et al. (1998), or Thatcher (1999).

In Table 1 we present the estimates for the Gompertz-Makeham model ($\mu(x) = \alpha e^{\beta x} + \gamma$) and the model outlined in Equation 2.[4]

Results

Relative Derivative Assuming that Equation 2 captures mortality dynamics correctly, the relative derivative of the force of mortality $\mu(x)$ is:

$$\frac{\frac{d\mu(x)}{dx}}{\mu(x)} = \frac{d\log(\mu(x))}{dx} = \frac{\frac{\alpha\beta\, e^{\beta x}}{1+\alpha\, e^{\beta x}} - \frac{\alpha^2\,\beta\, e^{2\beta x}}{\left(1+\alpha\, e^{\beta x}\right)^2}}{\frac{\alpha\, e^{\beta x}}{\alpha\, e^{\beta x}+1} + \gamma} \tag{3}$$

The age, x_1^*, when the relative derivative reaches its maximum is where its second derivative crosses 0:[5]

$$x_1^* = \frac{\log\left(\frac{\sqrt{\frac{\gamma}{1+\gamma}}}{\alpha}\right)}{\beta} \tag{4}$$

Figure 2 illustrates our findings for the life-table aging rate (i.e. the relative derivative of the force of mortality). The upper panel depicts with black + symbols observed death rates for women in England & Wales in 2004. The blue and green lines refer to a Gompertz-Makeham (blue) and logistic (green) mortality model fitted to the data. The lower panel plots the relative derivative of the force of mortality (assuming a logistic model). We added vertical red reference lines to both panels to indicate the age when the relative derivate reaches its maximum.

As known from many publications on the life-table aging rate (e.g. Horiuchi and Coale, 1990; Horiuchi and Wilmoth, 1998), the shape of the relative derivative of the force of mortality is a parabola and reaches its maximum in our case at age 66.

Speed Assuming that Equation 2 captures mortality dynamics correctly, the first derivative (i.e. the speed) of the force of mortality $\mu(x)$ is:

[4] Since they were not necessary, we do not include standard errors for the Gompertz Makeham model here.

[5] And, of course, its third derivative at this age must be negative.

$$\mu'(x) = \frac{d\mu(x)}{dx} = \frac{\alpha\beta e^{\beta x}}{1+\alpha e^{\beta x}} - \frac{\alpha^2\beta e^{2\beta x}}{(1+\alpha e^{\beta x})^2} \tag{5}$$

The age, x_2^*, when the first derivative reaches its maximum is:

$$x_2^* = -\frac{\log\alpha}{\beta} \tag{6}$$

The main difference between Figure 2 and Figure 3 is that the latter plots in the lower panel the shape of the first derivative of the (logistic) force of mortality and adds a reference line in both panels to indicate the age of maximum speed. In our empirical example the maximum absolute speed is attained at about age 104.

Acceleration Assuming that Equation 2 captures mortality dynamics correctly, the second derivative (i.e. the acceleration) of the force of mortality $\mu(x)$ is:

$$\mu''(x) = \frac{\partial^2\mu(x)}{\partial x^2} = \frac{\alpha\beta^2 e^{\beta x}}{1+\alpha e^{\beta x}} - \frac{3\alpha^2\beta^2 e^{2\beta x}}{(1+\alpha e^{\beta x})^2} + \frac{2\alpha^3\beta^2 e^{3\beta x}}{(1+\alpha e^{\beta x})^3} \tag{7}$$

The age, x_3^*, when the first derivative reaches its maximum is:

$$x_3^* = \frac{\log\left(-\frac{\sqrt{3}-2}{\alpha}\right)}{\beta} \tag{8}$$

Figure 4 plots in its lower panel the second derivative of the force of mortality and denotes the age of maximum acceleration with a vertical red line. We find that this age of 93 years is located at a prominent position in the upper panel, namely, when the logistic fit diverges from the Gompertz-Makeham fit.

Deviation from Gompertz-Makeham Figure 5 is similar to the upper panels of the previous figures. Here, we added 99% confidence bands around the point estimate of the logistic fit. The age when mortality deceleration sets in is when the logistic fit is significantly lower than the Gompertz-Makeham fit, i.e. when the Gompertz-Makeham point estimates are not anymore included in the confidence bands of the logistic fit. In our empirical case study, this age of 92.82 years is very close to the age of maximum acceleration.

Summary

Our paper presents four different ways to measure mortality deceleration. We propose two new methods (maximum acceleration, significant deviation from Gompertz-Makeham shape) and contrast them with previously employed ones (relative derivative, speed). Our two new methods find ages of mortality deceleration which are close to each other. Our choice of a logistic mortality shape might have influenced that the age of onset of mortality deceleration of the two previously employed methods are not located at prominent positions on the mortality curve. We suggest, however, that our two newly proposed methods should be considered as real alternatives to measure deceleration in the future due to their intuitive theoretical appeal and our empirical findings presented in Figures 4 and 5.

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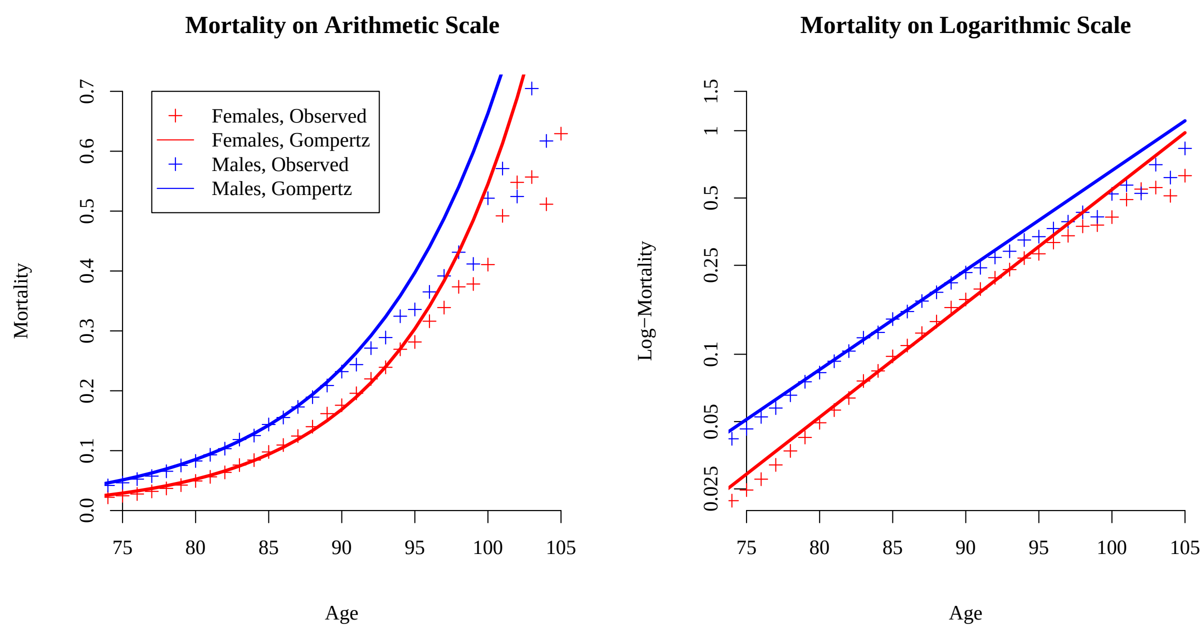
Tables and Figures

Table 1: Estimates for Fitting a Gompertz and a Logistic Model to Female Mortality Data from England & Wales in 2004

Model	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$
Gompertz-Makeham	3.0330e-06	1.2076e-01	1.2526e-03
Logistic	3.5985e-06	1.2062e-01	1.0285e-04
(Stand. Errors)	(1.0579e-07)	(3.4870e-04)	(3.4904e-05)

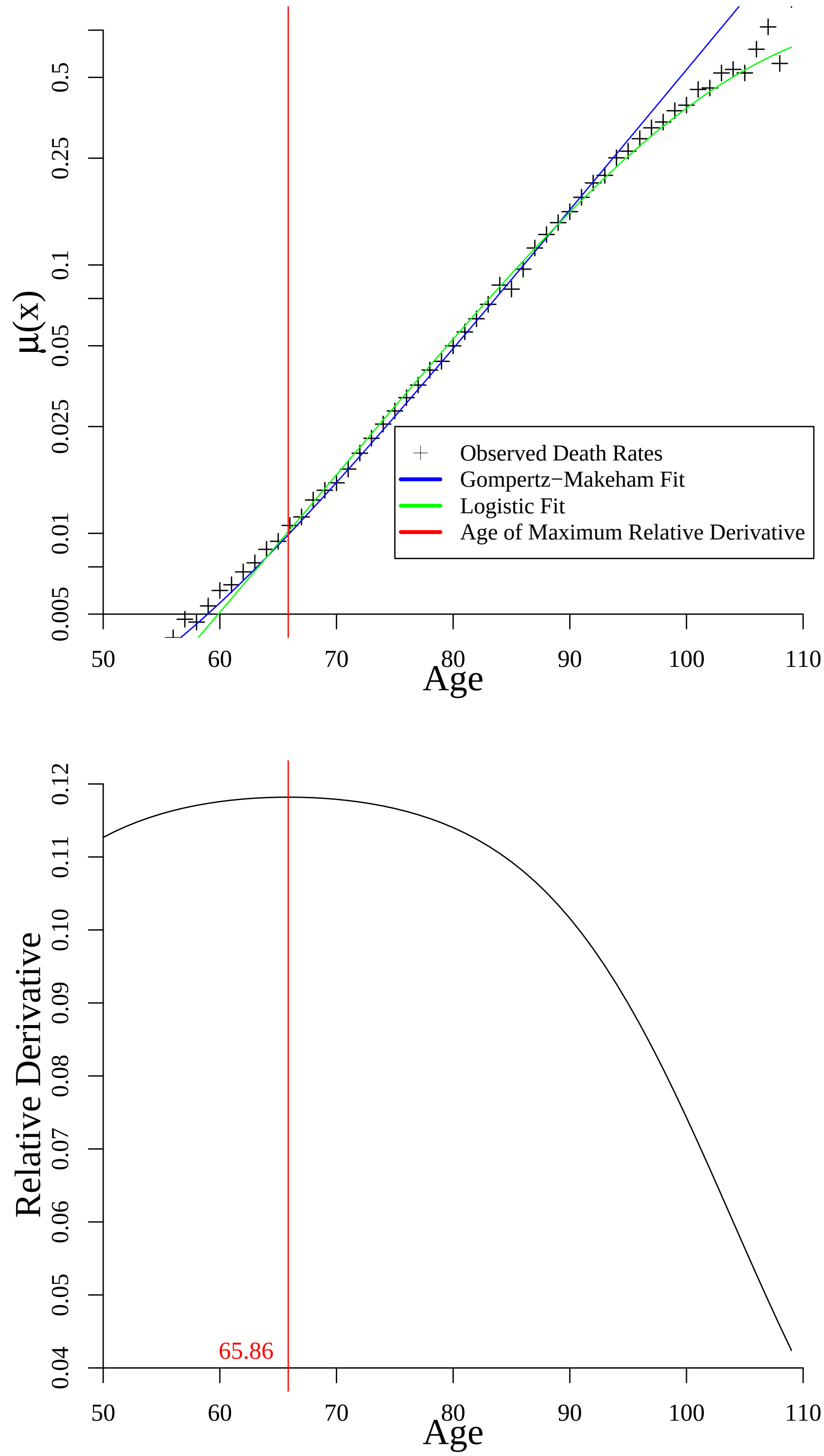
Figure 1: Fitting a Gompertz Model to Observed Mortality in Japan for Females and Males in 1990



Data Source: University of California, Berkeley (USA), and Max Planck Institute for Demographic Research, Rostock, (Germany) (2008)

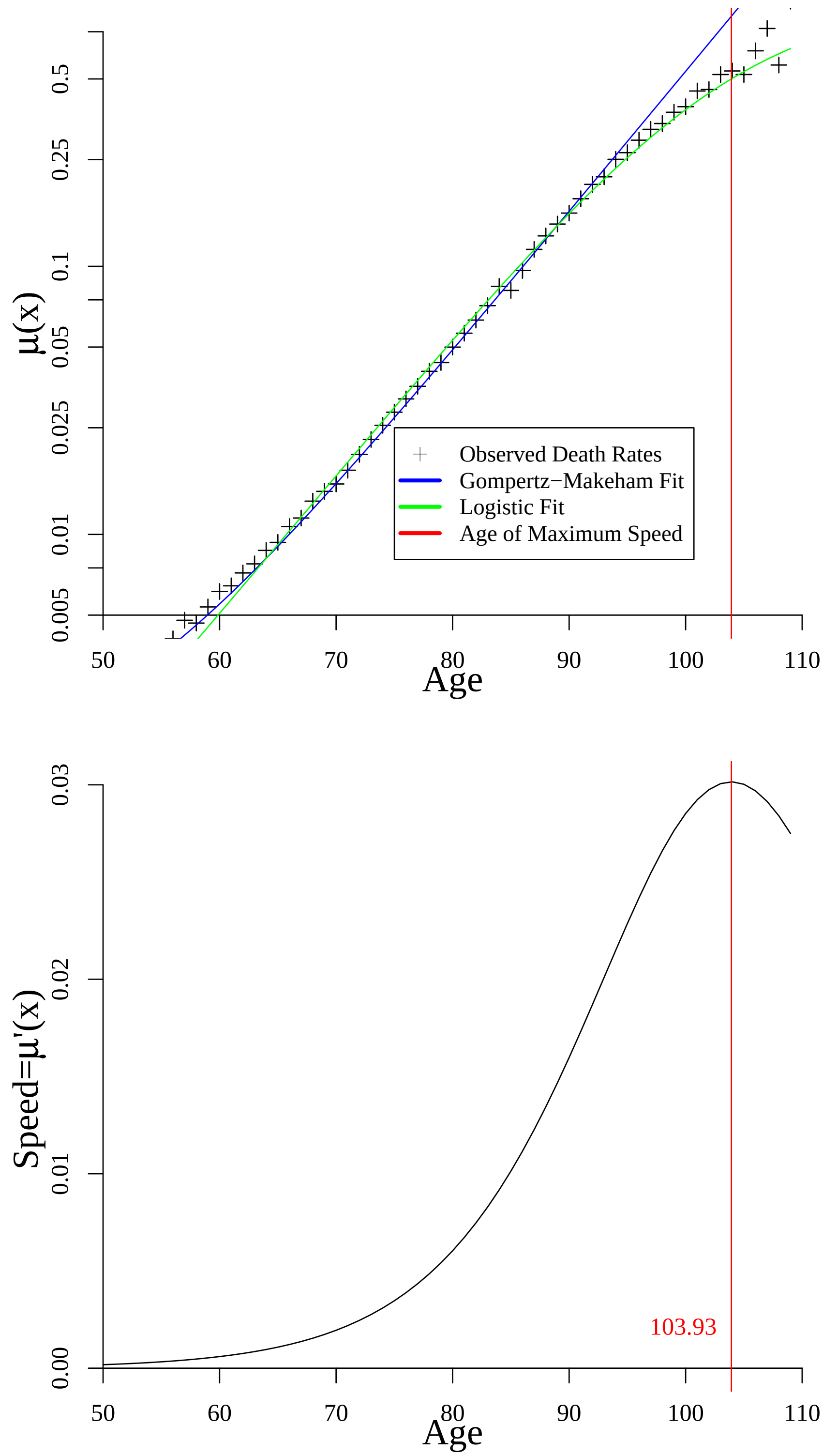
Parameter Estimates:	α	β	Log-Likelihood
Females:	4.184736e-06	1.171888e-01	1589700
Males:	2.223338e-05	1.025210e-01	1840096

Figure 2: Upper Panel: Observed and Fitted Mortality; Lower Panel: Relative Derivative of Logistic Fit of Mortality; Red Vertical Reference Lines are Added in Both Panels to Indicate the Age of the Maximum of the Relative Derivative



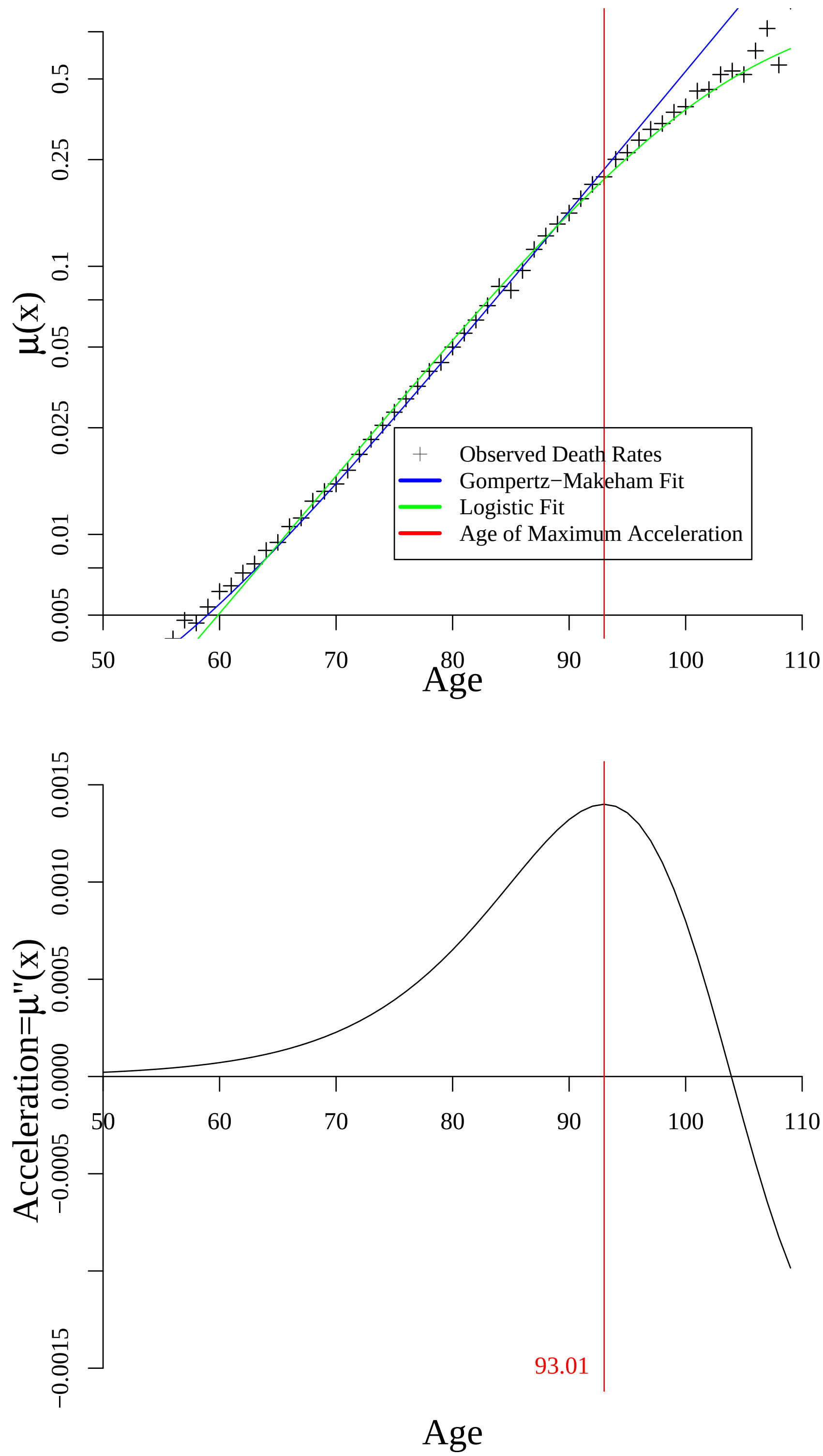
The presented results are based on data for females in England & Wales in 2004 obtained from the Human Mortality Database.

Figure 3: Upper Panel: Observed and Fitted Mortality; Lower Panel: First Derivative of Logistic Fit of Mortality (=Speed); Red Vertical Reference Lines are Added in Both Panels to Indicate the Age of Maximum Speed



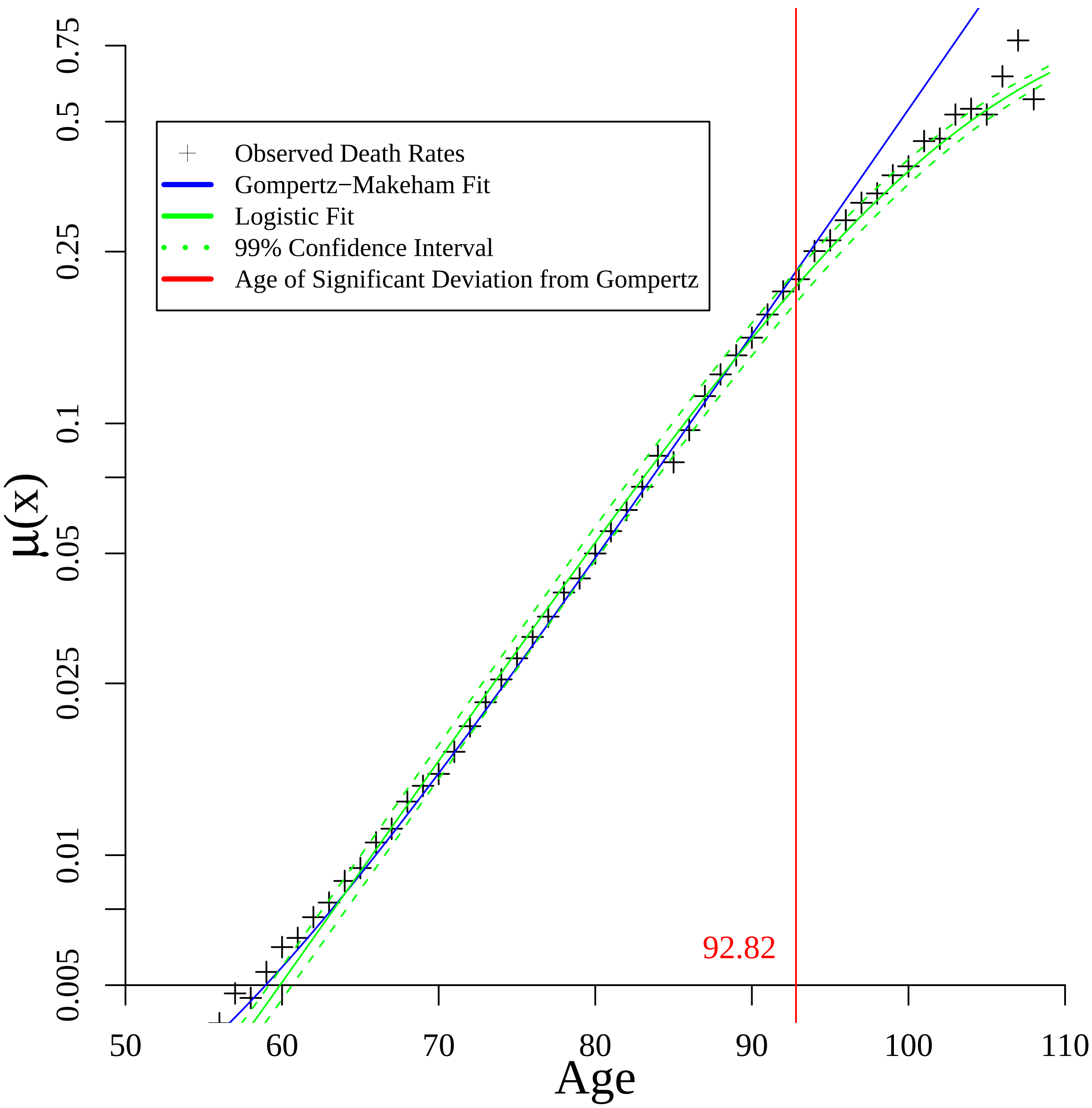
The presented results are based on data for females in England & Wales in 2004 obtained from the Human Mortality Database.

Figure 4: Upper Panel: Observed and Fitted Mortality; Lower Panel: Second Derivative of Logistic Fit of Mortality (=Acceleration); Red Vertical Reference Lines are Added in Both Panels to Indicate the Age of Maximum Acceleration



The presented results are based on data for females in England & Wales in 2004 obtained from the Human Mortality Database.

Figure 5: Observed Mortality; Gompertz Mortality Fit; Logistic Mortality Fit and Its 99% Confidence Interval; Age when Logistic Model Significantly Deviates from Gompertz Fit



The presented results are based on data for females in England & Wales in 2004 obtained from the Human Mortality Database.