

# Estimation of the foreigner's fertility in Italy using mathematical functions

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## 1. Introduction

In the study of immigration, the fertility of the foreigners in the host country is surely one of the most interesting topics that can be investigated especially for its contribution to the rise of total fertility in the receiving country. The data about foreigners currently available in Italy allow to promptly study the most important demographic phenomena, but difficulties arise at a level of single nationalities because of noteworthy lack of data. This is a first attempt to estimate age specific fertility rates of some foreign nationalities in Italy.

## 2. Data and methods

The available data are the total births distributed by age of the mother for the first four nationality in size order (Albanians, Moroccans, Romanians and Chinese) and for two smaller nationalities which have, however, interesting reproductive behaviors (Ecuadorians and Nigerians). The age structure of women by nationality is not given. Knowing the total number of births by nationality and the age distribution of women it could have been possible to estimate the specific fertility rates using, for instance, the De Simoni method (1982) based on the assumption of typical age specific rates distributions at different levels of fertility. The method here proposed, however, overpasses these difficulties using fertility theoretical curves (Duchêne, Gillet-de Stefano, 1974), which depend on three demographic indices - the total fertility, the average age at childbearing and the variance around this average age - a method already used in other past applications for different intents (Lombardo, De Bartolo, 1981).

The theoretical curves used are: Beta, Gamma, Lognormal, Third-degree Polynomial, Hadwiger, from which to choose the best fitting function.

Table 1 shows the starting data. The total fertility rate (TFR) and the mean age at childbearing ( $\bar{x}$ ) were accessible from the Italian National Institute of Statistics (Ministry of Interior, Italy, 2007), the variance related to this mean age ( $\sigma_x^2$ ) was estimated.

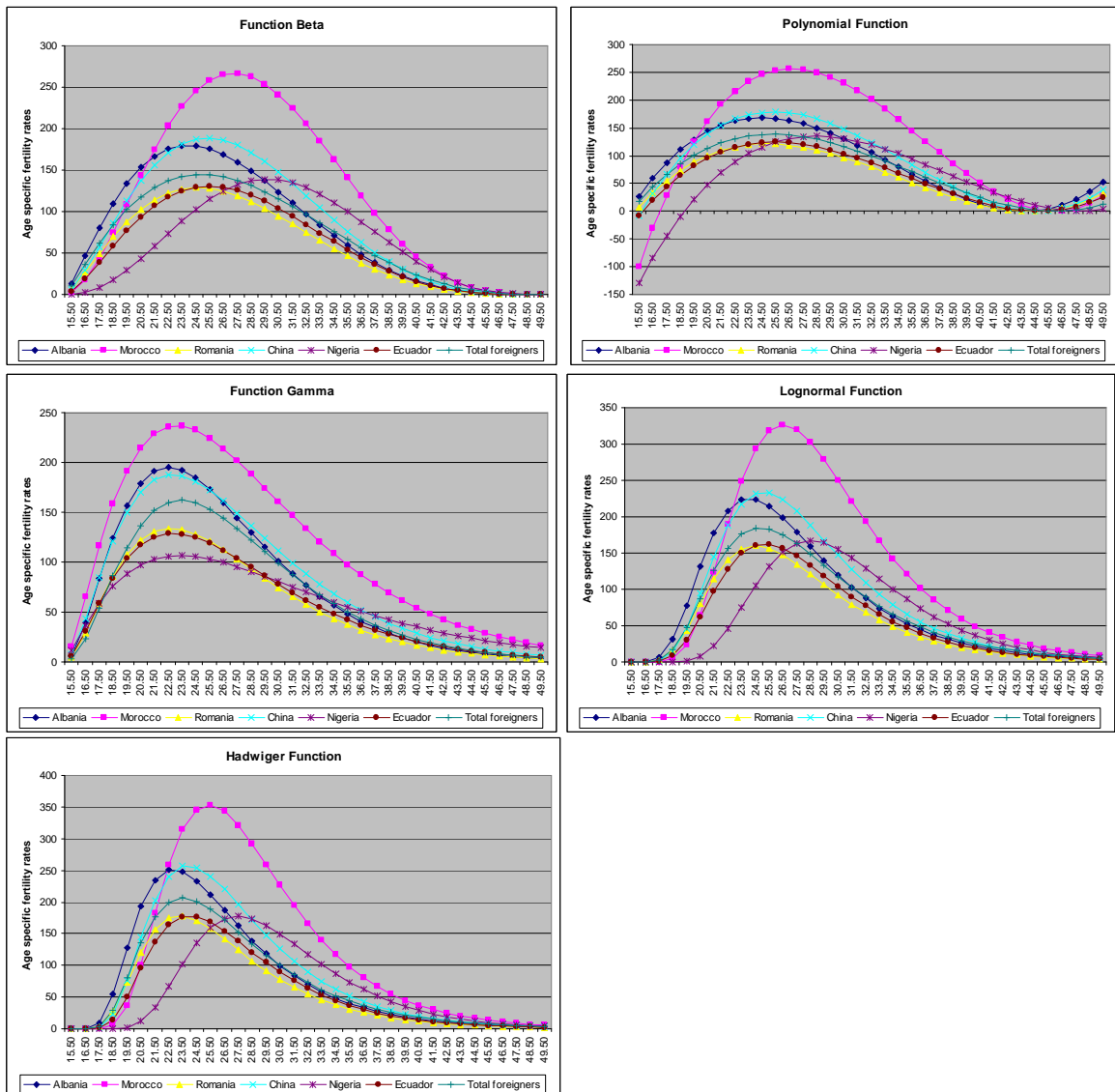
**Table 1:** Available and estimated fertility indicators for some foreign nationalities

| Nationalities           | TFR         | $\bar{x}$    | $\sigma_x^2$       |
|-------------------------|-------------|--------------|--------------------|
| Albania                 | 2,75        | 26,40        | 33,64031448        |
| Morocco                 | 4,19        | 28,60        | 33,64133644        |
| Romania                 | 1,98        | 26,80        | 33,64057075        |
| China                   | 2,92        | 27,30        | 33,64223756        |
| Nigeria                 | 2,20        | 30,40        | 33,64400906        |
| Ecuador                 | 2,03        | 27,40        | 33,64050062        |
| <b>Total foreigners</b> | <b>2,43</b> | <b>27,35</b> | <b>38,82654738</b> |

### 3. Main results

Figure 1 shows the main results of the application for each nationality and for the total foreign population.

**Figure 1:** Estimated age specific fertility rates for foreigners using the mathematical functions



With the aim of appreciate the level of adaptation of the used mathematical functions, Table 2 shows a comparison between the real values of the total fertility rate for the six nationalities, and those estimated using the age specific fertility rated calculated through our application. As can be seen the divergence between the values obtained by using the proposed method are quite modest. The same comparison has been made with the values of average age at childbearing, as shown in Table 3, putting evidence as there is almost no difference with the real distribution by age.

**Table 2:** *Real and estimated total fertility rate for some foreign nationalities*

|                         | <i>Real</i> | Beta        | Polynomial  | Gamma       | Lognormal   | Hadwiger    |
|-------------------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Albania                 | 2,75        | 2,75        | 2,87        | 2,73        | 2,72        | 2,74        |
| Morocco                 | 4,19        | 4,19        | 4,11        | 4,09        | 4,15        | 4,17        |
| Romania                 | 1,98        | 1,98        | 2,05        | 1,96        | 1,96        | 1,97        |
| China                   | 2,92        | 2,92        | 2,99        | 2,89        | 2,89        | 2,91        |
| Nigeria                 | 2,20        | 2,20        | 1,94        | 2,08        | 2,17        | 2,18        |
| Ecuador                 | 2,03        | 2,03        | 2,07        | 2,01        | 2,01        | 2,02        |
| <b>Total foreigners</b> | <b>2,43</b> | <b>2,43</b> | <b>2,45</b> | <b>2,41</b> | <b>2,40</b> | <b>2,41</b> |

**Table 3:** *Real and estimated average age at childbearing for some foreign nationalities*

|                         | <i>Real</i>  | Beta         | Polynomial   | Gamma        | Lognormal    | Hadwiger     |
|-------------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Albania                 | 26,40        | 26,40        | 27,35        | 26,24        | 27,51        | 26,27        |
| Morocco                 | 28,60        | 28,60        | 29,24        | 27,87        | 29,52        | 28,46        |
| Romania                 | 26,80        | 26,80        | 27,56        | 26,58        | 27,87        | 26,67        |
| China                   | 27,30        | 27,30        | 27,91        | 26,97        | 28,33        | 27,17        |
| Nigeria                 | 30,40        | 30,40        | 32,38        | 28,82        | 31,17        | 30,23        |
| Ecuador                 | 27,40        | 27,40        | 27,99        | 27,05        | 28,42        | 27,27        |
| <b>Total foreigners</b> | <b>27,35</b> | <b>27,35</b> | <b>27,53</b> | <b>27,12</b> | <b>28,42</b> | <b>27,15</b> |

#### 4. Final remarks

From the comparison between the real and the estimated values of the total fertility rate and the average age at childbearing, it is possible to appreciate how the mathematical function which best fit the age distribution of the fertility rates is the Beta one, although almost all show very little divergence from the real values.

While long-term observation have revealed that the Polynomial can be useful to describe the specific rates at high level of fertility (De Bartolo, 1983), other function, for instance Beta and Gamma, have proved to be suitable when fertility decreases (*ibidem*). The use of Polynomial function needs to be revised since it gives some negative values for two nationalities (Morocco and Nigeria) for ages before 19: these values, which surely cannot exist, can be determined by the low absolute values of the starting data and from the particular form of this function.

Surely, as data will become more reliable, the proposed procedure could give better results; also, it will be possible to take into consideration other functions than those proposed here and to try to estimate the fertility curves of other Italian immigrant populations.

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