# Principal Components Analysis of ASFR : 

# Application to the Recent Fertility Schedules around the World 

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## 1. The Purpose of this Analysis

The Statistical Technique of Principal Components Analysis is a very useful technique for examining whether only a few (One or Two or Three) components (which are assumed to be linear combinations of ' $p$ ' variables for which ' $n$ ' observations are available) could explain most of the variations in the observed set of ' $n$ ' values of the ' $p$ ' variables. These few components may then be used as representing the set of observed values for other purposes (Kendall and Stuart, 1976). Applying this technique to the set of Five Year Age Group ASFRs (Age Specific Fertility Rates) in the countries around the World for the years around 1960 (74 countries), around 1970 ( 68 countries) and around 1980 ( 53 countries), it was found by Sivamurthy (1985; 1986) that the first Three Principal Components (PCs) explained more than 95 percent of the variations in the observed values of ASFR.

Since the fertility situations in the countries around the World have changed greatly during the recent decades, it is thought that it would be useful and interesting to examine whether the same is true for the recent fertility schedules observed around the World. Hence, the purpose of the present analysis is to examine whether for the recent ASFR schedules for the years around the year 2001, the first Three components will explain more than 95 percent of the variations in the observed ASFRs.

Based on this analysis, the PC model proposed by Sivamurthy (1986) to represent the ASFR schedule, is revised. A new approach to fit the model to a given ASFR schedule is presented, and is applied to fit the revised model to the ASFR schedule of India, 1971.

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## 2. Technique of Principal Components Analysis

Let $\mathrm{X} 1, \mathrm{X} 2, \ldots, \mathrm{X} 7$ represent the ASFR values in the 7 age groups (15-19), (20-24), ..., (45-49) years. Then, from Statistics theory it is known that a set of 7 independent components U1, U2, ...., U7 can be obtained as linear functions of the observed Xi's such that the observed values of the Xi's could be reproduced by linear combinations of the Ui's (Kendall and Stuart, 1976).

Thus, $\mathrm{Ui}=\mathrm{b}(\mathrm{i} 1) \cdot \mathrm{X} 1+\mathrm{b}(\mathrm{i} 2) \cdot \mathrm{X} 2+\ldots+\mathrm{b}(\mathrm{i} 7) \cdot \mathrm{X} 7$
These Ui's are called Structural Components, and b(ij) are called Structural Coefficients.
If we consider the standardized variables: $x i=(X i-M X i) / S X i$ where $M X i$ and SXi are the mean and Standard Deviation of the ' $n$ ' observed values of the Xi , and we assume that the SXi is equal to the standard deviation in the statistical universe, then the $b$ (ij) could be estimated as the elements of the Latent Vector corresponding to the i-th Latent Root 'ri' of the matrix of correlation coefficients between the Xi's, normalized by the Square root of 'ri'. It should be noted that the sum of all the latent roots of the correlation matrix will be equal to ' p ' ( $=7$ in the present analysis) which is the total variance of all the standardized variables xi's. Thus the value of any particular latent root divided by ' $p$ ' gives the proportion of the variance of the Xi's accounted for by that particular component. It is customary to arrange the latent roots by the diminishing order of magnitude of this proportion. This facilitates the identification of the most dominant structural components. From the cumulative value of this proportion we can decide how many components could explain most of the variation in the values of the Xi's.

Then, the structural equation for the xi's can be written as follows :
xi = V1i.U1+V2i.U2+.....+V7i.U7 ... (2)
where $\mathrm{V}(\mathrm{ji})$ are called factor loadings that give the weights with which the components appear in the different xi variables, and the Uj's are called structural components. The application of the technique of Principal Components Analysis gives the values of $\mathrm{V}(\mathrm{ji})$ and the values of the latent roots from the observed matrix of correlation coefficients between the Xi's.

Since it has been observed in a previous analysis (Sivamurthy, 1986) that the first Three components accounted for more than 95 percent of the total variance in the observed Xi values, the following structural equation has been suggested by Sivamurthy (1986) as the Principal Components model ( Sivamurthy's PC model ) to represent the ASFR schedule (in fact, in the earlier paper he has suggested to take only Two components which explained about 90 percent of the variation in the Xi's):
$X i=M X i+(V 1 i . S X i) \cdot A 1+(V 2 i . S X i) \cdot A 2+(V 3 i . S X i) \cdot A 3$
where $\mathrm{A} 1, \mathrm{~A} 2$, \& A 3 are taken as the parameters of the model.
Using different values for A1, A2, \& A3 we can obtain different sets of ASFR values. In fact, Sivamurthy has given a set of numerical Tables of the model ASFR schedules using different A1 \& A2 values. These are useful for fertility estimation and projection. However, such an attempt is not made here since the ASFR schedules can be developed without much difficulty by using the computers.

The First component is the most dominant component and is taken as the indicator of the level of fertility in a population. Then, the Second component may be taken as the component which indicates the postponement of fertility as its effect is seen to be relatively more at the younger ages (< 25 years of age), and the Third component may be taken as the component indicating
the fertility limitation (control ) in the population as its effect seems to be relatively more at the older ages ( $>35$ years of age). Second and Third components together will determine the age pattern of fertility. With this interpretation, the Three components will have very interesting and useful practical meanings. It may also be more meaningful, in the present day controlled fertility situations, to specify the age patterns of fertility (say, for fertility projections) in terms of the proportion of fertility contributed by younger age cohorts ( <25 years) and in terms of the fertility contributed by older age cohorts (>35 years). Accordingly, a simple method is given in this paper for estimating the parameters of the model in terms of these two proportions (represented in this paper as $\mathrm{p} 1 \& \mathrm{p} 3$ ), along with the value of TFR (the total fertility rate).

## 3. Observed ASFR schedules used for the Analysis

The observed ASFR schedules for the 88 countries for which the basic data were available for the years considered in the present analysis, are taken from the United Nations Demographic Year Book, 2003. The schedules refer to the Five years around 2001 (i.e. the years 1999 to 2003). It is assumed that the schedules are reliable. (See Appendix Table A. 2 for the ASFR data used in the analysis).

The means and standard deviations (SDs) for the different age groups are presented in Table 1. The maximum and the minimum of the values of the ASFRs are also given in the table for showing the large variations in the respective ASFR values. It may be observed from Table 1 that except in the last two age groups (especially in the last age group), the variations in the ASFRs are quite large. The means and SDs are very small for the last two age groups $(40-44) \&(45-49)$ years, and are comparatively small for the age groups (15-19) \& (35-39) years. The ASFR in the last two age groups indicate that the practice of fertility control at ages greater than 40 years seems to have become a universal phenomenon.

Table 1 : Means and Standard Deviations (SD's) of the ASFRs (Xi s) of 88 Countries around the World for the Years 2001 (i.e.1999-2003)

|  | X1 | X2 | X3 | X4 | X5 | X6 | X7 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| N of cases | 88 | 88 | 88 | 88 | 88 | 88 | 88 |
| Minimum | 2.700 | 22.200 | 51.200 | 29.600 | 10.200 | 1.800 | 0.000 |
| Maximum | 111.800 | 211.200 | 242.900 | 201.600 | 128.300 | 60.100 | 22.100 |
| Mean | 30.411 | 90.311 | 111.159 | 88.727 | 44.666 | 12.251 | 1.577 |
| Standard Dev | 24.968 | 43.796 | 37.823 | 34.578 | 25.353 | 12.139 | 3.699 |

The Product Moment correlation coefficients between the ASFRs in the different age groups, are given in Table 2. It may be seen from the table that the correlation coefficients in the successive higher age groups are quite large ranging from 0.66 to 0.92 , as it was observed in the earlier analyses (Sivamurthy, 1985; 1986). An interesting point to be noted is that the fertility performance in the first two younger age groups (15-19) \& (20-24) has a much less correlation with the fertility performance in the next higher age groups as compared to the fertility performance in other age groups.

Table 2 : Product Moment Correlation Coefficients between ASFRs of the $\mathbf{8 8}$ Countries

|  | X 1 | X 2 | X 3 | X 4 | X |  | X |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| X 1 | 1.000 |  |  |  |  |  |  |
| X 2 | 0.699 | 1.000 |  |  |  |  |  |
| X | 0.304 | 0.663 | 1.000 |  |  |  |  |
| X | 0.069 | 0.272 | 0.808 | 1.000 |  |  |  |
| X | 0.306 | 0.439 | 0.765 | 0.918 |  |  |  |
| X 6 | 0.405 | 0.547 | 0.725 | 0.756 | 0.909 |  | 1.000 |

## 4. The Results of the Principal Components Analysis

The results of the Principal Components Analysis of the correlation matrix given in Table 2, are presented in Table 3 in a summary form.

Table 3 : Latent Roots (Eigen values) and Component Loadings for the Correlation Matrix

| Components | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Latent Roots (Eigen values) | 4.519 | 1.330 | 0.586 | 0.398 | 0.106 | 0.052 | 0.011 |
| Component Loadings |  |  |  |  |  |  |  |
| X1 | 0.468 | -0.802 | -0.004 | 0.348 | 0.131 | 0.012 | 0.000 |
| X2 | 0.684 | -0.630 | 0.112 | -0.303 | -0.162 | 0.063 | 0.015 |
| X3 | 0.880 | 0.042 | 0.353 | -0.262 | 0.151 | -0.091 | -0.027 |
| X4 | 0.830 | 0.459 | 0.262 | 0.118 | 0.051 | 0.108 | 0.055 |
| X5 | 0.926 | 0.241 | 0.069 | 0.240 | -0.133 | 0.028 | -0.068 |
| X6 | 0.947 | 0.078 | -0.233 | 0.106 | -0.090 | -0.147 | 0.043 |
| X7 | 0.784 | 0.116 | -0.567 | -0.185 | 0.106 | 0.070 | -0.014 |
| Percent of Total Variance |  |  |  |  |  |  |  |
| Explained \% | 64.55 | 19.00 | 8.37 | $5 . .69$ | 1.51 | 0.73 | 0.15 |
| Cumulative | 64.55 | 83.55 | 91.92 | 97.61 | 99.12 | 99.85 | 100.00 |

These indicate that in spite of the substantial changes in the fertility conditions in most of the countries around the World, the first Three Principal Components still explained about 92 percent of the total variation in the observed ASFR values of the 88 countries for the years around 2001 (i.e. 1999-2003). This is about 5 percentage points less than the percentage explained by the Three Principal Components for the ASFRs of the years around 1960 (for 74 countries), around 1970 (for 68 countries) and around 1980 (for 53 countries), (see Table 4 for the comparison). It may also be seen from Table 4 that the importance of the First component has decreased and that of the Second and the Third components has increased over time from 1970 to the recent time period of 2001. It may be said therefore that with the decreasing of fertility levels in the countries around the World, the age patterns of fertility have changed substantially in this period of time.

Table 4 : Comparison of Coefficients for the Structural Components representing ASFR at Different Time Periods

| Time Periods and Components | Value of Roots | Structural Coefficients for the Components |  |  |  |  |  |  | Percent <br> Variance | Ratio to |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 15-19 | 20-24 | 25-29 | 30-34 | 35-39 | 40-44 | 45-49 |  | First root |
|  |  | X1 | X2 | X3 | X4 | X5 | X6 | X7 | Explained |  |
| 1960 (74) |  |  |  |  |  |  |  |  |  |  |
| Components |  |  |  |  |  |  |  |  |  |  |
| First | 5.446 | 0.683 | 0.865 | 0.939 | 0.951 | 0.948 | 0.919 | 0.839 | 77.8 | - |
| Second | 0.846 | 0.697 | 0.378 | -0.115 | -0.241 | -0.266 | -0.271 | 0.041 | 89.9 | 0.155 |
| Third | 0.447 | 0.052 | -0.225 | -0.272 | -0.141 | -0.059 | 0.194 | 0.508 | 96.3 | 0.082 |
| 1970 (68) |  |  |  |  |  |  |  |  |  |  |
| Components |  |  |  |  |  |  |  |  |  |  |
| First | 5.553 | 0.646 | 0.896 | 0.936 | 0.967 | 0.977 | 0.961 | 0.805 | 79.3 | - |
| Second | 0.786 | 0.721 | 0.311 | -0.126 | -0.125 | -0.103 | -0.151 | -0.324 | 90.6 | 0.142 |
| Third | 0.434 | 0.216 | -0.155 | -0.285 | -0.167 | -0.091 | 0.137 | 0.476 | 96.8 | 0.078 |
| 1980 (53) |  |  |  |  |  |  |  |  |  |  |
| Components |  |  |  |  |  |  |  |  |  |  |
| First | 5.158 | 0.619 | 0.802 | 0.846 | 0.949 | 0.981 | 0.918 | 0.842 | 73.7 | - |
| Second | 1.089 | 0.721 | 0.529 | 0.011 | -0.121 | -0.151 | -0.302 | -0.402 | 89.2 | 0.211 |
| Third | 0.558 | 0.237 | 0.027 | -0.515 | -0.243 | -0.014 | 0.246 | 0.341 | 97.2 | 0.108 |
| General |  |  |  |  |  |  |  |  |  |  |
| Total(195) |  |  |  |  |  |  |  |  |  |  |
| Components |  |  |  |  |  |  |  |  |  |  |
| First | 5.411 | 0.675 | 0.875 | 0.922 | 0.958 | 0.971 | 0.931 | 0.783 | 77.3 | - |
| Second | 0.848 | 0.645 | 0.402 | 0.021 | -0.092 | -0.142 | -0.289 | -0.397 | 89.4 | 0.157 |
| Third | 0.515 | 0.323 | -0.051 | -0.347 | -0.232 | -0.116 | 0.138 | 0.451 | 96.8 | 0.095 |
| 2001 (88) |  |  |  |  |  |  |  |  |  |  |
| Components |  |  |  |  |  |  |  |  |  |  |
| First | 4.519 | 0.468 | 0.684 | 0.881 | 0.831 | 0.926 | 0.947 | 0.784 | 64.6 | - |
| Second | 1.331 | -0.802 | -0.631 | 0.042 | 0.459 | 0.241 | 0.078 | 0.116 | 83.6 | 0.294 |
| Third | 0.586 | -0.004 | 0.112 | 0.353 | 0.262 | 0.069 | -0.233 | -0.567 | 92.1 | 0.131 |

Note :- The earlier PC Model suggested by Sivamurthy (1987) for representing an ASFR schedule was a Two parameter model - the General Model (based on 195 countries ASFR schedules). Two more models : Early Marriage and Late Marriage models were also given.

Therefore, a revised version of Sivamurthy's PC model is suggested as a Three parameter model and is given here ( in Table 5) as a good Statistical model for representing an ASFR schedule. The model will be useful for fertility estimation and projection (Sivamurthy, 1986). A new method for estimating the parameters of the model is given below.

Table 5: Revised Version of Sivamurthy's PC Model for representing ASFR Schedule: 2001

| ASFR | Means <br> (MXi) | $\begin{gathered} \text { SD } \\ (\mathrm{SXi}) \end{gathered}$ | I <br> Component <br> (V1i) | V1i * Sxi | II <br> Component <br> (V2i) | V2i * Sxi | III <br> Component <br> (V3i) | V3i * Sxi |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| x1 | 30.411 | 24.968 | 0.468 | 11.6850 | - 0.802 | -20.0243 | -0.004 | - 0.0999 |
| x2 | 90.311 | 43.796 | 0.684 | 29.9565 | - 0.630 | -27.5915 | 0.112 | 4.9052 |
| x3 | 111.159 | 37.823 | 0.88 | 33.2842 | 0.042 | 1.5886 | 0.353 | 13.3515 |
| x4 | 88.727 | 34.578 | 0.83 | 28.6997 | 0.459 | 15.8713 | 0.262 | 9.0594 |
| x5 | 44.666 | 25.353 | 0.926 | 23.4769 | 0.241 | 6.1101 | 0.069 | 1.7494 |
| X6 | 12.251 | 12.139 | 0.947 | 11.4956 | 0.078 | 0.9468 | -0.233 | - 2.8284 |
| X7 | 1.577 | 3.699 | 0.784 | 2.900016 | 0.116 | 0.429084 | -0.567 | -2.0973 |
| Total=X1+ | TM $=379.102$ | - | - | $\mathrm{T} 1=141.4979$ | - | T2 $=-22.6699$ | - | T3 $=24.0399$ |
| X1+X2 | TM (1) $=120.722$ | - | - | $\mathrm{T} 1(1)=41.6415$ | - | T2 $(1)=-47.6158$ | - | $\mathrm{T} 3(1)=4.8053$ |
| X3+X4 | TM (2) $=199.886$ | - | - | $\mathrm{T} 1(2)=61.9839$ | - | $\mathrm{T} 2(2)=17.4599$ | - | T3(2)=22.4109 |
| X5+ | TM $(3)=58.494$ | - | - | $\mathrm{T} 1(3)=37.8725$ | - | $\mathrm{T} 2(3)=7.4860$ | - | $\mathrm{T} 3(3)=-3.1763$ |

Then, Xi can be computed using the equation (3) given in Section 2 above, after estimating the parameters A1, A2, A3.

## 5. A Simple Method for fitting the Model to a given ASFR Schedule

Fitting the model to a given ASFR schedule involves the estimation of the parameters of the model. Since there are Three parameters to be estimated in the revised version of Sivamurthy's PC Model presented in this paper, the following three independent criteria are used in the estimation procedure. The procedure is based on the Statistical method of estimating the parameters of a model using partial summation. The criteria suggested are:
(1) The total of the estimated ASFR $=$ The total of the given ASFR (i.e. TFR, the Total Fertility Rate),
(2) Proportion of TFR contributed by the age groups (15-19) \& (20-24) in the Model ASFR = Proportion of TFR contributed by the same age groups in the given ASFR (say, p1),
(3) Proportion of TFR contributed by the age groups (35+) years in the Model ASFR = Proportion of TFR contributed by the same age groups in the given ASFR (say, p3 ).

Using the representation of the Model given by Equation (3) in Section 2 above and equating the observed and expected TFR, we have :

$$
\begin{equation*}
\Sigma(\mathrm{Xi})=\sum(\mathrm{MXi})+\mathrm{A} 1 \cdot \sum(\mathrm{~V} 1 \mathrm{i} . \mathrm{SXi})+\mathrm{A} 2 . \sum(\mathrm{V} 2 \mathrm{i} . \mathrm{SXi})+\mathrm{A} 3 . \sum(\mathrm{V} 3 \mathrm{i} . \mathrm{SXi}) \tag{4}
\end{equation*}
$$ where $\sum$ denotes the summation over $\mathrm{i}=1,2, \ldots 7$.

For convenience, we write Equation (4) as follows (see Table 5 for explanation of T's) :

$$
\begin{equation*}
\mathrm{To}=\mathrm{TM}+\mathrm{A} 1 \cdot \mathrm{~T} 1+\mathrm{A} 2 \cdot \mathrm{~T} 2+\mathrm{A} 3 \cdot \mathrm{~T} 3 \tag{5}
\end{equation*}
$$

The other two criteria given above can be written as follows :

Substituting the value of A1 from Equation (5) into Equations (6) \& (7) and simplifying we have :

$$
\begin{align*}
& \mathrm{A} 1=[(\mathrm{To}-\mathrm{TM})-\mathrm{A} 2 \cdot \mathrm{~T} 2-\mathrm{A} 3 \cdot \mathrm{~T} 3] /(\mathrm{T} 1)  \tag{8}\\
& \text { A2 . [T2(1)-T2 . (T1(1)/T1 )] = \{[p1-T1(1)/T1]. (To - TM) - [TM(1) - TM . p1] } \\
& \text { - A3.[T3(1)-T3. (T1(1) / T1)] \} . . . (9) } \\
& \text { A3 . [T3(3)-T3 . (T1(3) / T1 )] = \{ [p3-T1(3) / T1] . (To - TM) - [TM(3) - TM . p3] } \\
& \text { - A2 . [T2(3)-T2 . (T1(3) / T1)] \} }
\end{align*}
$$

Now, substituting the numerical values for the T's , p1 and p3 in Equations (9) \& (10) we estimate the parameters $A 2 \& A 3$ and then $A 1$ is estimated by substituting the estimated $A 2 \& A 3$ in Equation (8).

A numerical application of this method is given in Table A. 1 which shows the results of the fitting of the revised Sivamurthy's PC Model to the ASFR schedule of India, 1971. Estimates of these ASFR using other Models are also presented in the table for comparison. It may be seen from Table A. 1 that the fit of the new model is good except in the last age group.

## 6. Conclusions and Suggestions

The analysis has brought out that the first three Principal Components explained 92 percent of the total variance in the ASFRs of the 88 countries around the World for the years around 2001 (i.e.1999-2003) for which the basic data were available in the United Nations Demographic Year Book of 2003. Based on this result a revised version of Sivamurthy's PC model for representing an ASFR schedule, is presented. This will be useful for estimating and projecting ASFR values for any population.

From the method of fitting of the model given in Section 5 above, it is apparent that specifications for the future fertility projections may be given in terms of TFR, p1and p3 which will be particularly useful since the practice of fertility control seems to have become a universal phenomenon. This kind of specification will also be interesting in examining the impact of certain fertility policies. Also, we would like to suggest that if ASFR are required by single years of age for any research, projection or other purposes, we should first obtain Five Year Age group ASFR using the method given in this paper and then use a linear interpolation as it is usually done in the construction of Non-Reproductive Life Tables (see Sivamurthy \& Sivamurthy, Chetna M. 2009).

Further, in view of the fact that the institution of marriage is becoming somewhat loose in practice in recent years, it seems to be important to study the male ASFRs also in addition to studying the female ASFRs and use the equal dominance Two Sex model (see Sivamurthy, 1982a ) for population projections in the future.

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## Appendix Table A. 1 :

Fitting the Revised Sivamurthy's PC Model to the ASFR Schedule of India, 1971


Note :- (a) Parameters A1, A2, \& A3 are estimated using the T's values given in Table 5 and the values of p1 = 0.3175 and p3 $=0.2416$ computed from Col.(2) for India 1971.
(b) Computed from marital fertility rates given in Sivamurthy (1982b) and adjusted to obtain the observed TFR of India 1971.

Age Specific Fertility Rates (ASFR) for 88 Countries around the World for the Years 1999-2003

| Countries / Years | Age Groups |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 15-19 | 20-24 | 25-29 | 30-34 | 35-39 | 40-44 | 45-49 | TFR |
| 1. Egypt 1999 | 18.5 | 192.8 | 226.3 | 162.8 | 87.7 | 25.7 | 6.7 | 3.60 |
| 2. Mauritious 2003 | 37.1 | 109.7 | 110.6 | 71.8 | 35.8 | 8.7 | 0.6 | 1.87 |
| 3. Morocco 2001 | 28.9 | 88.3 | 102.3 | 104.4 | 73.6 | 32.1 | 7.6 | 2.19 |
| 4. Namibia 2001 | 51.2 | 135.8 | 144.6 | 137.3 | 103.1 | 59.8 | 22.1 | 3.27 |
| 5. Reunion 1999 | 35.1 | 115.9 | 143.0 | 107.1 | 61.3 | 15.1 | 1.3 | 2.39 |
| 6. Anguilla 2001 | 61.7 | 136.3 | 102.2 | 49.8 | 37.7 | 15.6 | 0.0 | 2.02 |
| 7. Bermuda 2000 | 25.5 | 81.1 | 83.4 | 97.3 | 51.3 | 5.1 | 0.0 | 1.72 |
| 8. Canada 2002 | 15.2 | 54.0 | 95.4 | 89.4 | 36.1 | 6.2 | 0.2 | 1.48 |
| 9. Costa Rica 2003 | 69.4 | 114.2 | 111.9 | 77.3 | 35.9 | 9.7 | 0.9 | 2.10 |
| 10.Cuba 2003 | 48.3 | 97.9 | 90.1 | 60.0 | 24.5 | 4.7 | 0.2 | 1.63 |
| 11. El Salvador 2003 | 78.6 | 117.5 | 95.1 | 67.4 | 43.5 | 16.6 | 2.4 | 2.11 |
| 12. Greenland 2000 | 60.9 | 154.8 | 126.0 | 75.9 | 35.9 | 7.3 | 0.6 | 2.31 |
| 13. Grenada 2000 | 54.7 | 107.0 | 132.7 | 113.8 | 64.1 | 26.7 | 1.1 | 2.50 |
| 14. Guadeloupe 2003 | 25.9 | 81.3 | 132.3 | 108.6 | 64.6 | 19.3 | 0.8 | 2.16 |
| 15. Guatemala 1999 | 110.3 | 211.2 | 193.4 | 162.6 | 128.3 | 60.1 | 13.8 | 4.40 |
| 16. Jamaica 2003 | 69.7 | 111.2 | 90.3 | 76.6 | 47.8 | 17 | 0.9 | 2.07 |
| 17. Martinique 2003 | 26.2 | 67.7 | 116.7 | 99.2 | 57.2 | 16.7 | 0.7 | 1.92 |
| 18. Panama 2000 | 95.8 | 148.7 | 128.2 | 94.8 | 50.8 | 14.2 | 1.8 | 2.67 |
| 19. Puerto Rico 2003 | 60.6 | 112.7 | 91.3 | 58.3 | 24.4 | 4.7 | 0.2 | 1.76 |
| 20Saint Kitts and Nevis 2000 | 86.6 | 140.2 | 123.1 | 80.0 | 57.4 | 20.0 | 1.0 | 2.54 |
| 21. Saint Vincent 2000 | 73.0 | 128.5 | 100.1 | 80.0 | 72.8 | 16.5 | 0.6 | 2.36 |
| 22. U S A 2002 | 43.7 | 103.6 | 113.6 | 91.5 | 41.4 | 8.3 | 0.5 | 2.01 |
| 23. Argentina 2003 | 59.1 | 115.1 | 117.6 | 107.7 | 61.6 | 18.5 | 1.4 | 2.40 |
| 24. Chile 2003 | 50.3 | 87.5 | 94.8 | 81.7 | 47.3 | 13.4 | 0.7 | 1.88 |
| 25. French Guiana 2003 | 111.8 | 201.8 | 236.5 | 144.0 | 84.7 | 27.3 | 2.1 | 4.04 |
| 26. Suriname 2000 | 68.3 | 150.1 | 148.2 | 104.3 | 56.2 | 16.0 | 1.8 | 2.72 |
| 27. Uruguay 2002 | 65.5 | 103.2 | 107.0 | 90.0 | 49.7 | 13.9 | 0.9 | 2.15 |
| 28. Venezuela 2002 | 84.8 | 129.0 | 109.9 | 80.1 | 42.9 | 14.1 | 2.9 | 2.32 |
| 29. Armenia 2003 | 29.3 | 126.6 | 71.5 | 29.6 | 10.2 | 2.4 | 0.2 | 1.35 |
| 30. Azerbaijan 2003 | 27.7 | 126.4 | 95.3 | 44.9 | 18.7 | 4.9 | 0.4 | 1.59 |
| 31. Brunei 2001 | 29.9 | 90.9 | 125.0 | 108.1 | 68.0 | 24.5 | 1.8 | 2.24 |
| 32. China: Hong Kong 2003 | 3.6 | 28.0 | 51.2 | 49.4 | 23.3 | 4.1 | 0.2 | 0.80 |
| 33. China: Macao 2003 | 4.4 | 27.7 | 53.9 | 54.0 | 22.9 | 4.4 | 0.1 | 0.84 |
| 34. Cyprus 2003 | 6.6 | 56.2 | 106.8 | 81.3 | 32.4 | 7.4 | 0.7 | 1.46 |
| 35. Georgia 2003 | 33.8 | 94.0 | 75.4 | 47.4 | 19.0 | 5.4 | 0.4 | 1.38 |
| 36. Israel 2003 | 15.5 | 113.6 | 180.9 | 163.7 | 90.8 | 22.6 | 1.9 | 2.94 |
| 37. Japan 2003 | 5.7 | 37.0 | 88.4 | 85.1 | 32.2 | 4.5 | 0.1 | 1.27 |
| 38. Kazakhstan 2003 | 25.8 | 136.1 | 120.2 | 76.4 | 38.1 | 8.0 | 0.5 | 2.03 |
| 39. Korea 2002 | 2.7 | 26.8 | 116.2 | 79.2 | 16.9 | 2.5 | 0.2 | 1.22 |
| 40. Kyrgyzstan 2003 | 28.5 | 165.1 | 144.7 | 96.0 | 51.1 | 15.5 | 3.5 | 2.52 |
| 41. Maldives 2003 | 14.5 | 116.3 | 133.4 | 103.2 | 63.4 | 15.4 | 2.8 | 2.25 |
| 42. Mongolia 2003 | 18.6 | 123.1 | 119.5 | 80.7 | 38.2 | 14.3 | 4.4 | 1.99 |
| 43. Pakistan 2001 | 24.2 | 162.0 | 242.9 | 197.2 | 118.5 | 57.9 | 21.9 | 4.12 |
| 44.Philippines 2000 | 31.6 | 139.2 | 163.1 | 132.4 | 86.3 | 36.1 | 6.2 | 2.97 |
| 45. Singapore 2003 | 7.0 | 33.7 | 85.4 | 94.5 | 37.8 | 6.3 | 0.2 | 1.32 |
| 46. Uzbekistan 2000 | 21.1 | 205.4 | 161.4 | 89.7 | 31.5 | 7.0 | 0.8 | 2.58 |
| 47. Austria 2003 | 13.2 | 57.1 | 94.5 | 73.5 | 30.4 | 5.9 | 0.3 | 1.37 |
| 48. Belarus 2003 | 23.3 | 93.2 | 73.1 | 37.0 | 12.2 | 2.2 | 0.1 | 1.21 |

## Appendix Table A. 2

 (Continued)| 49. Bulgaria 2003 | 40.4 | 80.5 | 75.3 | 37.2 | 11.3 | 1.8 | 0.1 | 1.23 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50. Croatia 2003 | 14.0 | 66.9 | 93.1 | 64.2 | 23.0 | 4.2 | 0.2 | 1.33 |
| 51. Czech 2003 | 11.4 | 53.8 | 94.4 | 57.4 | 17.6 | 3.1 | 0.1 | 1.19 |
| 52. Denmark 2003 | 6.0 | 46.4 | 125.6 | 121.2 | 46.6 | 7.7 | 3 | 1.78 |
| 53. Estonia 2002 | 21.9 | 76.4 | 88.6 | 58.0 | 24.3 | 4.9 | 0.1 | 1.37 |
| 54. Finland 2003 | 10.4 | 57.0 | 115.5 | 106.9 | 49.4 | 10.8 | 0.5 | 1.75 |
| 55. France 2002 | 8.1 | 55.9 | 129.4 | 117.7 | 51.7 | 11.4 | 0.6 | 1.87 |
| 56. Germany 2003 | 11.7 | 50.5 | 86.3 | 78.4 | 33.5 | 5.5 | 0.2 | 1.33 |
| 57. Gibraltar 2001 | 24.9 | 77.5 | 168.3 | 110.8 | 43.4 | 7.9 | 0.0 | 2.16 |
| 58. Greece 2003 | 11.1 | 43.7 | 82.3 | 77.7 | 34.9 | 6.3 | 0.7 | 1.28 |
| 59. Hungary 2003 | 20.9 | 56.4 | 88.8 | 62.0 | 23.3 | 4.0 | 0.1 | 1.28 |
| 60. Iceland 2003 | 16.2 | 75.9 | 130.3 | 115.7 | 48.5 | 11.8 | 0.4 | 1.99 |
| 61. Ireland 2003 | 18.8 | 50.9 | 92.5 | 133.4 | 81.2 | 15.5 | 0.4 | 1.96 |
| 62. Italy 2003 | 7.1 | 35.7 | 78.1 | 84.6 | 41.9 | 7.7 | 0.4 | 1.28 |
| 63. Latvia 2002 | 16.0 | 72.6 | 80.3 | 51.2 | 21.1 | 4.9 | 0.4 | 1.23 |
| 64. Liechtenstein 2003 | 8.8 | 32.2 | 91.9 | 90.1 | 40.0 | 5.9 | 0.8 | 1.35 |
| 65. Lithuania 2003 | 20.5 | 76.0 | 85.2 | 46.4 | 19.1 | 4.2 | 0.2 | 1.26 |
| 66. Luxembourg 2003 | 11.1 | 57.5 | 103.4 | 102.5 | 43.9 | 8.3 | 0.4 | 1.64 |
| 67. Malta 2003 | 15.9 | 53.4 | 101.1 | 81.6 | 29.1 | 5.2 | 0.1 | 1.43 |
| 68. Netherlands 2003 | 7.1 | 41.3 | 109.6 | 131.3 | 52.9 | 7.0 | 0.3 | 1.75 |
| 69. Norway 2002 | 10.1 | 59.5 | 121.0 | 109.3 | 44.1 | 7.7 | 0.2 | 1.76 |
| 70. Poland 2003 | 14.5 | 64.1 | 88.1 | 52.9 | 20.9 | 4.6 | 0.2 | 1.23 |
| 71. Portugal 2003 | 20.1 | 51.2 | 89.7 | 84.6 | 35.7 | 7.1 | 0.4 | 1.44 |
| 72. Repub. Moldova 2003 | 29.2 | 89.7 | 67.6 | 40.7 | 13.8 | 2.6 | 0.1 | 1.22 |
| 73. Romania 2003 | 34.0 | 79.7 | 79.0 | 41.7 | 16.1 | 3.1 | 0.2 | 1.27 |
| 74. Russian Federation 2001 | 27.6 | 94.6 | 70.6 | 39.0 | 13.4 | 2.4 | 0.1 | 1.24 |
| 75. San Marino 2003 | 8.0 | 22.2 | 66.4 | 91.5 | 50.0 | 9.0 | 1.0 | 1.24 |
| 76. Serbia 2001 | 25.3 | 105.7 | 113.6 | 66.3 | 25.9 | 5.1 | 0.4 | 1.71 |
| 77. Slovakia 2002 | 21.5 | 68.6 | 83.4 | 46.0 | 16.2 | 2.9 | 0.1 | 1.19 |
| 78. Slovenia 2003 | 5.8 | 44.3 | 94.8 | 70.7 | 21.8 | 3.5 | 0.1 | 1.20 |
| 79. Spain 2001 | 10.0 | 27.4 | 65.5 | 95.1 | 45.8 | 7.4 | 0.4 | 1.26 |
| 80. Sweden 2002 | 6.6 | 47.7 | 109.2 | 110.7 | 47.3 | 8.9 | 0.3 | 1.65 |
| 81. Switzerland 2002 | 5.4 | 40.7 | 89.9 | 94.5 | 40.4 | 6.8 | 0.4 | 1.39 |
| 82. Yugoslav 2003 | 25.4 | 112.1 | 127.9 | 64.4 | 19.7 | 3.4 | 0.2 | 1.77 |
| 83. Ukraine 2003 | 28.6 | 93.6 | 67.3 | 33.1 | 10.7 | 2.0 | 0.1 | 1.18 |
| 84. U. K. 2003 | 26.6 | 70.1 | 95.9 | 94.5 | 45.9 | 9.1 | 0.5 | 1.71 |
| 85. Australia 2003 | 16.1 | 53.8 | 102.8 | 112.7 | 54.4 | 10.0 | 0.5 | 1.75 |
| 86. New Caledonia 2003 | 20.3 | 109.2 | 129.5 | 118.9 | 64.8 | 18.3 | 0.3 | 2.31 |
| 87. New Zealand 2003 | 26.2 | 68.9 | 110.2 | 114.5 | 59.3 | 12.2 | 0.6 | 1.96 |
| 88. Tonga 1999 | 28.3 | 128.3 | 220.2 | 201.6 | 128.2 | 49.3 | 3.2 | 3.80 |

Source :- United Nations (2006) U.N. Demographic Year Book 2003, Regular Issue, Statistics Division, United Nations, New York.

