Principal Components Analysis of ASFR :

Application to the Recent Fertility Schedules around the World

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1. The Purpose of this Analysis

The Statistical Technique of Principal Components Analysis is a very useful technique for examining whether only a few (One or Two or Three) components (which are assumed to be linear combinations of 'p' variables for which 'n' observations are available) could explain most of the variations in the observed set of 'n' values of the 'p' variables. These few components may then be used as representing the set of observed values for other purposes (Kendall and Stuart, 1976). Applying this technique to the set of Five Year Age Group ASFRs (Age Specific Fertility Rates) in the countries around the World for the years around 1960 (74 countries), around 1970 (68 countries) and around 1980 (53 countries), it was found by Sivamurthy (1985; 1986) that the first Three Principal Components (PCs) explained more than 95 percent of the variations in the observed values of ASFR.

Since the fertility situations in the countries around the World have changed greatly during the recent decades, it is thought that it would be useful and interesting to examine whether the same is true for the recent fertility schedules observed around the World. Hence, the purpose of the present analysis is to examine whether for the recent ASFR schedules for the years around the year 2001, the first Three components will explain more than 95 percent of the variations in the observed ASFRs.

Based on this analysis, the PC model proposed by Sivamurthy (1986) to represent the ASFR schedule, is revised. A new approach to fit the model to a given ASFR schedule is presented, and is applied to fit the revised model to the ASFR schedule of India, 1971.

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2. Technique of Principal Components Analysis

Let X1, X2, ..., X7 represent the ASFR values in the 7 age groups (15-19), (20-24), ..., (45-49) years. Then, from Statistics theory it is known that a set of 7 independent components U1, U2, ..., U7 can be obtained as linear functions of the observed Xi's such that the observed values of the Xi's could be reproduced by linear combinations of the Ui's (Kendall and Stuart, 1976).

Thus, Ui = b(i1).X1+b(i2).X2+...+b(i7).X7 (1)

These Ui's are called Structural Components, and b(ij) are called Structural Coefficients.

If we consider the standardized variables: xi = (Xi - MXi)/SXi where MXi and SXi are the mean and Standard Deviation of the 'n' observed values of the Xi, and we assume that the SXi is equal to the standard deviation in the statistical universe, then the b(ij) could be estimated as the elements of the Latent Vector corresponding to the i-th Latent Root 'ri' of the matrix of correlation coefficients between the Xi's, normalized by the Square root of 'ri'. It should be noted that the sum of all the latent roots of the correlation matrix will be equal to 'p' (=7 in the present analysis) which is the total variance of all the standardized variables xi's. Thus the value of any particular latent root divided by 'p' gives the proportion of the variance of the Xi's accounted for by that particular component. It is customary to arrange the latent roots by the diminishing order of magnitude of this proportion. This facilitates the identification of the most dominant structural components. From the cumulative value of this proportion we can decide how many components could explain most of the variation in the values of the Xi's.

Then, the structural equation for the xi's can be written as follows :

xi = V1i.U1 + V2i.U2 + + V7i.U7 ... (2)

where V(ji) are called factor loadings that give the weights with which the components appear in the different xi variables, and the Uj's are called structural components. The application of the technique of Principal Components Analysis gives the values of V(ji) and the values of the latent roots from the observed matrix of correlation coefficients between the Xi's.

Since it has been observed in a previous analysis (Sivamurthy, 1986) that the first Three components accounted for more than 95 percent of the total variance in the observed Xi values, the following structural equation has been suggested by Sivamurthy (1986) as the Principal Components model (Sivamurthy's PC model) to represent the ASFR schedule (in fact, in the earlier paper he has suggested to take only Two components which explained about 90 percent of the variation in the Xi's):

Xi = MXi + (V1i.SXi).A1 + (V2i.SXi).A2 + (V3i.SXi).A3 (3) where A1, A2, & A3 are taken as the parameters of the model.

Using different values for A1, A2, & A3 we can obtain different sets of ASFR values. In fact, Sivamurthy has given a set of numerical Tables of the model ASFR schedules using different A1 & A2 values. These are useful for fertility estimation and projection. However, such an attempt is not made here since the ASFR schedules can be developed without much difficulty by using the computers.

The First component is the most dominant component and is taken as the indicator of the level of fertility in a population. Then, the Second component may be taken as the component which indicates the postponement of fertility as its effect is seen to be relatively more at the younger ages (< 25 years of age), and the Third component may be taken as the component indicating

the fertility limitation (control) in the population as its effect seems to be relatively more at the older ages (> 35 years of age). Second and Third components together will determine the age pattern of fertility. With this interpretation, the Three components will have very interesting and useful practical meanings. It may also be more meaningful , in the present day controlled fertility situations, to specify the age patterns of fertility (say, for fertility projections) in terms of the proportion of fertility contributed by younger age cohorts (<25 years) and in terms of the fertility contributed by older age cohorts (>35 years). Accordingly, a simple method is given in this paper for estimating the parameters of the model in terms of these two proportions (represented in this paper as p1 & p3), along with the value of TFR (the total fertility rate).

3. Observed ASFR schedules used for the Analysis

The observed ASFR schedules for the 88 countries for which the basic data were available for the years considered in the present analysis, are taken from the United Nations Demographic Year Book, 2003. The schedules refer to the Five years around 2001 (i.e. the years 1999 to 2003). It is assumed that the schedules are reliable. (See Appendix Table A.2 for the ASFR data used in the analysis).

The means and standard deviations (SDs) for the different age groups are presented in Table 1. The maximum and the minimum of the values of the ASFRs are also given in the table for showing the large variations in the respective ASFR values. It may be observed from Table 1 that except in the last two age groups (especially in the last age group), the variations in the ASFRs are quite large. The means and SDs are very small for the last two age groups (40-44) & (45-49) years, and are comparatively small for the age groups (15-19) & (35-39) years. The ASFR in the last two age groups indicate that the practice of fertility control at ages greater than 40 years seems to have become a universal phenomenon.

	X1	X2	X3	X4	X5	X6	X7
N of cases	88	88	88	88	88	88	88
Minimum	2.700	22.200	51.200	29.600	10.200	1.800	0.000
Maximum	111.800	211.200	242.900	201.600	128.300	60.100	22.100
Mean	30.411	90.311	111.159	88.727	44.666	12.251	1.577
Standard Dev	24.968	43.796	37.823	34.578	25.353	12.139	3.699

Table 1 : Means and Standard Deviations (SD's) of the ASFRs (Xi s)of 88 Countries around the World for the Years 2001 (i.e.1999-2003)

The Product Moment correlation coefficients between the ASFRs in the different age groups, are given in Table 2. It may be seen from the table that the correlation coefficients in the successive higher age groups are quite large ranging from 0.66 to 0.92, as it was observed in the earlier analyses (Sivamurthy, 1985; 1986). An interesting point to be noted is that the fertility performance in the first two younger age groups (15-19) & (20-24) has a much less correlation with the fertility performance in the next higher age groups as compared to the fertility performance in other age groups.

	X1	X2	X3	X4	X5	X6	X7
X1	1.000						
X2	0.699	1.000					
X3	0.304	0.663	1.000				
X4	0.069	0.272	0.808	1.000			
X5	0.306	0.439	0.765	0.918	1.000		
X6	0.405	0.547	0.725	0.756	0.909	1.000	
Х7	0.226	0.443	0.553	0.546	0.659	0.844	1.000

 Table 2 : Product Moment Correlation Coefficients between ASFRs of the 88 Countries

4. The Results of the Principal Components Analysis

The results of the Principal Components Analysis of the correlation matrix given in Table 2, are presented in Table 3 in a summary form.

Components	1	2	3	4	5	6	7
Latent Roots							
(Eigen values)	4.519	1.330	0.586	0.398	0.106	0.052	0.011
Component Loadings							
X1	0.468	- 0.802	- 0.004	0.348	0.131	0.012	0.000
X2	0.684	- 0.630	0.112	- 0.303	- 0.162	0.063	0.015
X3	0.880	0.042	0.353	- 0.262	0.151	- 0.091	- 0.027
X4	0.830	0.459	0.262	0.118	0.051	0.108	0.055
X5	0.926	0.241	0.069	0.240	- 0.133	0.028	- 0.068
X6	0.947	0.078	- 0.233	0.106	- 0.090	- 0.147	0.043
X7	0.784	0.116	- 0.567	- 0.185	0.106	0.070	- 0.014
Percent of Total Variance							
Explained %	64.55	19.00	8.37	569	1.51	0.73	0.15
Cumulative	64.55	83.55	91.92	97.61	99.12	99.85	100.00

Table 3 : Latent Roots (Eigen values) and Component Loadings for the Correlation Matrix

These indicate that in spite of the substantial changes in the fertility conditions in most of the countries around the World, the first Three Principal Components still explained about 92 percent of the total variation in the observed ASFR values of the 88 countries for the years around 2001 (i.e. 1999-2003). This is about 5 percentage points less than the percentage explained by the Three Principal Components for the ASFRs of the years around 1960 (for 74 countries), around 1970 (for 68 countries) and around 1980 (for 53 countries), (see Table 4 for the comparison). It may also be seen from Table 4 that the importance of the First component has decreased and that of the Second and the Third components has increased over time from 1970 to the recent time period of 2001. It may be said therefore that with the decreasing of fertility levels in the countries around the World, the age patterns of fertility have changed substantially in this period of time.

Time Periods	Value of		Structural C	Coefficients f	or the Comp	onents			Percent	Ratio to
and	Roots	15-19	20-24	25-29	30-34	35-39	40-44	45-49	Variance	First root
Components		X1	X2	X3	X4	X5	X6	X7	Explained	
<u>1960 (74)</u>										
Components										
First	5.446	0.683	0.865	0.939	0.951	0.948	0.919	0.839	77.8	-
Second	0.846	0.697	0.378	-0.115	-0.241	-0.266	-0.271	0.041	89.9	0.155
Third	0.447	0.052	-0.225	-0.272	-0.141	-0.059	0.194	0.508	96.3	0.082
1970 (68)										
Components										
First	5.553	0.646	0.896	0.936	0.967	0.977	0.961	0.805	79.3	-
Second	0.786	0.721	0.311	-0.126	-0.125	-0.103	-0.151	-0.324	90.6	0.142
Third	0.434	0.216	-0.155	-0.285	-0.167	-0.091	0.137	0.476	96.8	0.078
<u>1980 (53)</u>										
Components										
First	5.158	0.619	0.802	0.846	0.949	0.981	0.918	0.842	73.7	-
Second	1.089	0.721	0.529	0.011	-0.121	-0.151	-0.302	-0.402	89.2	0.211
Third	0.558	0.237	0.027	-0.515	-0.243	-0.014	0.246	0.341	97.2	0.108
<u>General</u>										
<u>Total(195)</u>										
Components										
First	5.411	0.675	0.875	0.922	0.958	0.971	0.931	0.783	77.3	-
Second	0.848	0.645	0.402	0.021	-0.092	-0.142	-0.289	-0.397	89.4	0.157
Third	0.515	0.323	-0.051	-0.347	-0.232	-0.116	0.138	0.451	96.8	0.095
<u>2001 (88)</u>										
Components										
First	4.519	0.468	0.684	0.881	0.831	0.926	0.947	0.784	64.6	_
Second	1.331	-0.802	-0.631	0.042	0.459	0.020	0.078	0.116	83.6	0.294
Third	0.586	-0.002	0.112	0.353	0.459	0.241	-0.233	-0.567	92.1	0.234
. mu	0.000	-0.004	0.112	0.000	0.202	0.009	-0.200	-0.007	52.1	0.101

Table 4 : Comparison of Coefficients for the Structural Components representing ASFR at Different Time Periods

Note :- The earlier PC Model suggested by Sivamurthy (1987) for representing an ASFR schedule was a Two parameter model – the **General Model** (based on 195 countries ASFR schedules).Two more models : Early Marriage and Late Marriage models were also given.

Therefore, a revised version of Sivamurthy's PC model is suggested as a Three parameter model and is given here (in Table 5) as a good Statistical model for representing an ASFR schedule. The model will be useful for fertility estimation and projection (Sivamurthy, 1986). A new method for estimating the parameters of the model is given below.

			1		11		III	
	Means	SD	Component		Component		Component	
ASFR	(MXi)	(SXi)	(V1i)	V1i * Sxi	(V2i)	V2i * Sxi	(V3i)	V3i * Sxi
x1	30.411	24.968	0.468	11.6850	- 0.802	- 20.0243	- 0.004	- 0.0999
x2	90.311	43.796	0.684	29.9565	- 0.630	- 27.5915	0.112	4.9052
x3	111.159	37.823	0.88	33.2842	0.042	1.5886	0.353	13.3515
x4	88.727	34.578	0.83	28.6997	0.459	15.8713	0.262	9.0594
x5	44.666	25.353	0.926	23.4769	0.241	6.1101	0.069	1.7494
X6	12.251	12.139	0.947	11.4956	0.078	0.9468	- 0.233	- 2.8284
X7	1.577	3.699	0.784	2.900016	0.116	0.429084	- 0.567	- 2.0973
Total=X1+	TM = 379.102	-	-	T1 = 141.4979	-	T2 = - 22.6699	-	T3 = 24.0399
X1+X2	TM(1)=120.722	-	-	T1(1)=41.6415	-	T2(1)= - 47.6158	-	T3(1)= 4.8053
X3+X4	TM(2)=199.886	-	-	T1(2)=61.9839	-	T2(2) = 17.4599	-	T3(2)= 22.4109
X5+	TM(3)= 58.494	-	-	T1(3)=37.8725	-	T2(3) = 7.4860	-	T3(3)= - 3.1763

Table 5: Revised Version of Sivamurthy's PC Model for representing ASFR Schedule: 2001

Then, Xi can be computed using the equation (3) given in Section 2 above, after estimating the parameters A1, A2, A3.

5. A Simple Method for fitting the Model to a given ASFR Schedule

Fitting the model to a given ASFR schedule involves the estimation of the parameters of the model. Since there are Three parameters to be estimated in the revised version of Sivamurthy's PC Model presented in this paper, the following three independent criteria are used in the estimation procedure. The procedure is based on the Statistical method of estimating the parameters of a model using partial summation. The criteria suggested are:

- (1) The total of the estimated ASFR = The total of the given ASFR (i.e. TFR, the Total Fertility Rate),
- (2) Proportion of TFR contributed by the age groups (15-19) & (20-24) in the Model ASFR = Proportion of TFR contributed by the same age groups in the given ASFR (say, p1),
- (3) Proportion of TFR contributed by the age groups (35+) years in the Model ASFR = Proportion of TFR contributed by the same age groups in the given ASFR (say, p3).

Using the representation of the Model given by Equation (3) in Section 2 above and equating the observed and expected TFR, we have :

$$\Sigma(Xi) = \Sigma (MXi) + A1. \Sigma (V1i.SXi) + A2. \Sigma (V2i.SXi) + A3. \Sigma (V3i.SXi) ... (4)$$

where Σ denotes the summation over i=1, 2, ... 7.

For convenience, we write Equation (4) as follows (see Table 5 for explanation of T's) : To = TM + A1 . T1 + A2 . T2 + A3. T3 (5)

The other two criteria given above can be written as follows :

$$p1 = \frac{To(1)}{To} = \frac{TM(1) + A1 \cdot T1(1) + A2 \cdot T2(1) + A3 \cdot T3(1)}{TM + A1 \cdot T1 + A2 \cdot T2 + A3 \cdot T3} \dots (6)$$

$$p3 = \frac{To(3)}{To} = \frac{TM(3) + A1 \cdot T1(3) + A2 \cdot T2(3) + A3 \cdot T3(3)}{TO} \qquad (7)$$

Substituting the value of A1 from Equation (5) into Equations (6) & (7) and simplifying we have :

$$A1 = [(To - TM) - A2 \cdot T2 - A3 \cdot T3] / (T1)$$

$$A2 \cdot [T2(1) - T2 \cdot (T1(1) / T1)] = \{ [p1 - T1(1) / T1] \cdot (To - TM) - [TM(1) - TM \cdot p1] \\ - A3 \cdot [T3(1) - T3 \cdot (T1(1) / T1)] \} . . . (9)$$

$$A3 \cdot [T3(3) - T3 \cdot (T1(3) / T1)] = \{ [p3 - T1(3) / T1] \cdot (To - TM) - [TM(3) - TM \cdot p3] \\ - A2 \cdot [T2(3) - T2 \cdot (T1(3) / T1)] \} . . . (10)$$

Now, substituting the numerical values for the T's , p1 and p3 in Equations (9) & (10) we estimate the parameters A2 & A3 and then A1 is estimated by substituting the estimated A2 & A3 in Equation (8).

A numerical application of this method is given in Table A.1 which shows the results of the fitting of the revised Sivamurthy's PC Model to the ASFR schedule of India, 1971. Estimates of these ASFR using other Models are also presented in the table for comparison. It may be seen from Table A.1 that the fit of the new model is good except in the last age group.

6. Conclusions and Suggestions

The analysis has brought out that the first three Principal Components explained 92 percent of the total variance in the ASFRs of the 88 countries around the World for the years around 2001 (i.e.1999-2003) for which the basic data were available in the United Nations Demographic Year Book of 2003. Based on this result a revised version of Sivamurthy's PC model for representing an ASFR schedule, is presented. This will be useful for estimating and projecting ASFR values for any population.

From the method of fitting of the model given in Section 5 above, it is apparent that specifications for the future fertility projections may be given in terms of TFR, p1and p3 which will be particularly useful since the practice of fertility control seems to have become a universal phenomenon. This kind of specification will also be interesting in examining the impact of certain fertility policies. Also, we would like to suggest that if ASFR are required by single years of age for any research, projection or other purposes, we should first obtain Five Year Age group ASFR using the method given in this paper and then use a linear interpolation as it is usually done in the construction of Non-Reproductive Life Tables (see Sivamurthy & Sivamurthy, Chetna M. 2009).

Further, in view of the fact that the institution of marriage is becoming somewhat loose in practice in recent years, it seems to be important to study the male ASFRs also in addition to studying the female ASFRs and use the equal dominance Two Sex model (see Sivamurthy, 1982a) for population projections in the future.

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Appendix Table A.1 :

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Age group	ameter Model	Coale's Model (fitted with One point procedure)								
V1i.SXi V2i.SXi V3i.SXi (esti- ing One least pola- mated) point squares ted (b) A1= 5.597 A2=- 0.317 A3=-1.841 A1=2.249 A1=1.979 A1=2.0 A2=- 0.231 A2=0.000 A2=0.0 (1) (2) (3) (4) (5) (6) (7) (8) (9) (10) (11) 15-19 95 30.411 65.401 6.348 0.184 102.3 99.7 104.0 104.3 146.2 20-24 265 90.311 167.666 8.746 - 9.029 257.7 263.3 268.8 269.4 262.2 25-29 271 111.159 186.292 - 0.504 -24.575 272.4 272.4 273.0 273.7 253.9 30-34 229 88.727 160.632 - 5.031 -16.675 227.6 223.8 219.5 220.3 220.0 35-39 159 44.666 131.400 - 1.937 - 3.220 170.9 166.1 159.4 160.1 166.4 40-44 78 12.251 64.341 - 0.300 + 5.206 81.5 79.3 72.9 73.3 76.1 45-49 37 1.577 16.231 - 0.136 + 3.860 21.6 30.4 36.2 26.3 9.8 TFR 5.67 5.67 5.67 5.67 5.64 5.67 Mean				A1*	A2*	A3*	ASFR	Fitted us-	Fitted b	y Inter-	• • • •
A1= $5.597 \ A2=-0.317 \ A3=-1.841$ A1= $2.249 \ A2=-0.231 \ A2=0.000 \ A2=0.0 \ A2=-0.231 \ A2=-0.001 \ A2=-0.001$		· · ·		V1i.SXi	V2i.SXi	V3i.SXi					M=0.1204
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$							mated)				(b)
(1) (2) (3) (4) (5) (6) (7) (8) (9) (10) (11) 15-19 95 30.411 65.401 6.348 0.184 102.3 99.7 104.0 104.3 146.2 20-24 265 90.311 167.666 8.746 - 9.029 257.7 263.3 268.8 269.4 262.2 25-29 271 111.159 186.292 - 0.504 -24.575 272.4 273.0 273.7 253.9 30-34 229 88.727 160.632 - 5.031 -16.675 227.6 223.8 219.5 220.3 220.0 35-39 159 44.666 131.400 - 1.937 - 3.220 170.9 166.1 159.4 160.1 166.4 40-44 78 12.251 64.341 - 0.300 + 5.206 81.5 79.3 72.9 73.3 76.1 45-49 37 1.577 16.231 - 0.136 + 3.860 21.6 30.4 36.2 26.3 9.8 TFR 5.67 5.67 5.67			A	1=5.597 A	2=- 0.317	A3=-1.841					
15-19 95 30.411 65.401 6.348 0.184 102.3 99.7 104.0 104.3 146.2 20-24 265 90.311 167.666 8.746 - 9.029 257.7 263.3 268.8 269.4 262.2 25-29 271 111.159 186.292 - 0.504 -24.575 272.4 273.0 273.7 253.9 30-34 229 88.727 160.632 - 5.031 -16.675 227.6 223.8 219.5 220.3 220.0 35-39 159 44.666 131.400 - 1.937 - 3.220 170.9 166.1 159.4 160.1 166.4 40-44 78 12.251 64.341 - 0.300 + 5.206 81.5 79.3 72.9 73.3 76.1 45-49 37 1.577 16.231 - 0.136 + 3.860 21.6 30.4 36.2 26.3 9.8 TFR 5.67 5.67 5.67 5.67 5.67 5.67 5.67	(1)	(2)	(2)	(1)	(5)	(6)					(11)
20-24 265 90.311 167.666 8.746 - 9.029 257.7 263.3 268.8 269.4 262.2 25-29 271 111.159 186.292 - 0.504 -24.575 272.4 273.0 273.7 253.9 30-34 229 88.727 160.632 - 5.031 -16.675 227.6 223.8 219.5 220.3 220.0 35-39 159 44.666 131.400 - 1.937 - 3.220 170.9 166.1 159.4 160.1 166.4 40-44 78 12.251 64.341 - 0.300 + 5.206 81.5 79.3 72.9 73.3 76.1 45-49 37 1.577 16.231 - 0.136 + 3.860 21.6 30.4 36.2 26.3 9.8 TFR 5.67 5.67 5.67 5.67 5.67 5.67 5.67	(1)	(2)	(3)	(4)	(5)	(0)	(7)	(0)	(9)	(10)	(11)
20-24 265 90.311 167.666 8.746 - 9.029 257.7 263.3 268.8 269.4 262.2 25-29 271 111.159 186.292 - 0.504 -24.575 272.4 273.0 273.7 253.9 30-34 229 88.727 160.632 - 5.031 -16.675 227.6 223.8 219.5 220.3 220.0 35-39 159 44.666 131.400 - 1.937 - 3.220 170.9 166.1 159.4 160.1 166.4 40-44 78 12.251 64.341 - 0.300 + 5.206 81.5 79.3 72.9 73.3 76.1 45-49 37 1.577 16.231 - 0.136 + 3.860 21.6 30.4 36.2 26.3 9.8 TFR 5.67 5.67 5.67 5.67 5.67 5.67 5.67											· · · · · · · · · · · · · · · · · · ·
25-29 271 111.159 186.292 - 0.504 -24.575 272.4 272.4 273.0 273.7 253.9 30-34 229 88.727 160.632 - 5.031 -16.675 227.6 223.8 219.5 220.3 220.0 35-39 159 44.666 131.400 - 1.937 - 3.220 170.9 166.1 159.4 160.1 166.4 40-44 78 12.251 64.341 - 0.300 + 5.206 81.5 79.3 72.9 73.3 76.1 45-49 37 1.577 16.231 - 0.136 + 3.860 21.6 30.4 36.2 26.3 9.8 TFR 5.67 5.67 5.67 5.67 5.67 5.67 5.67	15-19	95	30.411	65.401	6.348	0.184	102.3	99.7	104.0	104.3	146.2
30-34 229 88.727 160.632 - 5.031 -16.675 227.6 223.8 219.5 220.3 220.0 35-39 159 44.666 131.400 - 1.937 - 3.220 170.9 166.1 159.4 160.1 166.4 40-44 78 12.251 64.341 - 0.300 + 5.206 81.5 79.3 72.9 73.3 76.1 45-49 37 1.577 16.231 - 0.136 + 3.860 21.6 30.4 36.2 26.3 9.8 TFR 5.67 5.67 5.67 5.67 5.67 5.67 5.67	20-24	265	90.311	167.666	8.746	- 9.029	257.7	263.3	268.8	269.4	262.2
35-39 159 44.666 131.400 - 1.937 - 3.220 170.9 166.1 159.4 160.1 166.4 40-44 78 12.251 64.341 - 0.300 + 5.206 81.5 79.3 72.9 73.3 76.1 45-49 37 1.577 16.231 - 0.136 + 3.860 21.6 30.4 36.2 26.3 9.8 TFR 5.67 5.67 5.67 5.67 5.64 5.67 Mean Mean <td>25-29</td> <td>271</td> <td>111.159</td> <td>186.292</td> <td>- 0.504</td> <td>-24.575</td> <td>5 272.4</td> <td>272.4</td> <td>273.0</td> <td>273.7</td> <td>253.9</td>	25-29	271	111.159	186.292	- 0.504	-24.575	5 272.4	272.4	273.0	273.7	253.9
40-44 78 12.251 64.341 - 0.300 + 5.206 81.5 79.3 72.9 73.3 76.1 45-49 37 1.577 16.231 - 0.136 + 3.860 21.6 30.4 36.2 26.3 9.8 TFR 5.67 5.67 5.67 5.64 5.67 Mean 9 9 9 9 9	30-34	229	88.727	160.632	- 5.031	-16.675	5 227.6	223.8	219.5	220.3	220.0
45-49 37 1.577 16.231 - 0.136 + 3.860 21.6 30.4 36.2 26.3 9.8 TFR 5.67 5.67 5.67 5.67 5.67 5.67 Mean 5.67 5.67 5.67 5.67 5.67 5.67	35-39	159	44.666	131.400	- 1.937	'- 3.220	170.9	166.1	159.4	160.1	166.4
TFR 5.67 5.67 5.67 5.64 5.67 Mean	40-44	78	12.251	64.341	- 0.300	+ 5.206	81.5	79.3	72.9	73.3	76.1
Mean	45-49) 37	1.577	16.231	- 0.136	6 + 3.860	21.6	30.4	36.2	26.3	9.8
							5.67	5.67	5.67	5.64	5.67
							29.4	29.5	29.4	29.2	28.7
Chisquare	Chiso	uare									
All 7 age groups 12.70 2.12 1.64 5.88 95.40			ups				12.70	2.12	1.64	5.88	95.40
Middle 5 age groups 1.20 0.46 0.84 0.75 1.93							1.20	0.46			

Fitting the Revised Sivamurthy's PC Model to the ASFR Schedule of India, 1971

Note :- (a) Parameters A1, A2, & A3 are estimated using the T's values given in Table 5 and the values of p1 = 0.3175 and p3 = 0.2416 computed from Col.(2) for India 1971.
(b) Computed from marital fertility rates given in Sivamurthy (1982b) and adjusted to

obtain the observed TFR of India 1971.

Age Groups Countries / Years 15-19 20-24 25-29 35-39 40-44 45-49 TFR 30-34 1. Egypt 1999 18.5 192.8 226.3 162.8 87.7 25.7 6.7 3.60 2. Mauritious 2003 37.1 109.7 110.6 71.8 35.8 8.7 0.6 1.87 3. Morocco 2001 28.9 88.3 102.3 104.4 73.6 32.1 7.6 2.19 4. Namibia 2001 51.2 135.8 144.6 137.3 103.1 59.8 22.1 3.27 5. Reunion 1999 35.1 115.9 143.0 107.1 61.3 15.1 1.3 2.39 6. Anguilla 2001 61.7 136.3 102.2 49.8 37.7 15.6 0.0 2.02 7. Bermuda 2000 25.5 81.1 83.4 97.3 51.3 5.1 0.0 1.72 8. Canada 2002 1.48 15.2 54.0 95.4 89.4 36.1 6.2 0.2 9. Costa Rica 2003 9.7 69.4 114.2 111.9 77.3 35.9 0.9 2.10 10.Cuba 2003 48.3 97.9 90.1 60.0 24.5 4.7 0.2 1.63 11. El Salvador 2003 78.6 117.5 95.1 67.4 43.5 16.6 2.4 2.11 12. Greenland 2000 60.9 154.8 126.0 75.9 35.9 7.3 0.6 2.31 13. Grenada 2000 54.7 107.0 132.7 113.8 64.1 26.7 1.1 2.50 132.3 0.8 14. Guadeloupe 2003 25.9 81.3 108.6 64.6 19.3 2.16 15. Guatemala 1999 110.3 211.2 193.4 162.6 128.3 60.1 13.8 4.40 16. Jamaica 2003 69.7 111.2 90.3 76.6 47.8 17 0.9 2.07 17. Martinique 2003 26.2 67.7 116.7 99.2 57.2 16.7 0.7 1.92 18. Panama 2000 148.7 128.2 50.8 14.2 1.8 2.67 95.8 94.8 19. Puerto Rico 2003 60.6 112.7 91.3 58.3 24.4 4.7 0.2 1.76 20Saint Kitts and Nevis 2000 86.6 140.2 123.1 80.0 57.4 20.0 1.0 2.54 21. Saint Vincent 2000 73.0 128.5 100.1 80.0 72.8 16.5 0.6 2.36 22. U S A 2002 103.6 91.5 41.4 8.3 0.5 2.01 43.7 113.6 23. Argentina 2003 59.1 115.1 117.6 107.7 61.6 18.5 1.4 2.40 24. Chile 2003 50.3 87.5 94.8 81.7 47.3 13.4 0.7 1.88 2.1 25. French Guiana 2003 111.8 201.8 236.5 144.0 84.7 27.3 4.04 150.1 148.2 104.3 56.2 16.0 1.8 2.72 26. Suriname 2000 68.3 27. Uruguay 2002 65.5 103.2 107.0 90.0 49.7 13.9 0.9 2.15 28. Venezuela 2002 84.8 129.0 109.9 80.1 42.9 14.1 2.9 2.32 29. Armenia 2003 29.3 126.6 71.5 29.6 10.2 2.4 0.2 1.35 30. Azerbaijan 2003 27.7 126.4 95.3 44.9 18.7 4.9 0.4 1.59 31. Brunei 2001 29.9 90.9 125.0 108.1 68.0 24.5 1.8 2.24 32. China: Hong Kong 2003 3.6 28.0 51.2 49.4 23.3 4.1 0.2 0.80 33. China: Macao 2003 27.7 0.84 4.4 53.9 54.0 22.9 4.4 0.1 34. Cyprus 2003 6.6 56.2 106.8 81.3 32.4 7.4 0.7 1.46 35. Georgia 2003 33.8 94.0 75.4 47.4 19.0 5.4 0.4 1.38 36. Israel 2003 90.8 15.5 113.6 180.9 163.7 22.6 1.9 2.94 37. Japan 2003 5.7 37.0 88.4 85.1 32.2 4.5 0.1 1.27 38. Kazakhstan 2003 25.8 136.1 120.2 76.4 38.1 8.0 0.5 2.03 39. Korea 2002 2.7 26.8 116.2 79.2 16.9 2.5 0.2 1.22 40. Kyrgyzstan 2003 28.5 165.1 144.7 96.0 51.1 15.5 3.5 2.52 41. Maldives 2003 116.3 133.4 103.2 63.4 2.8 14.5 15.4 2.25 42. Mongolia 2003 18.6 123.1 119.5 80.7 38.2 14.3 4.4 1.99 43. Pakistan 2001 24.2 162.0 242.9 118.5 57.9 21.9 4.12 197.2 44.Philippines 2000 31.6 139.2 163.1 132.4 86.3 36.1 6.2 2.97 45. Singapore 2003 7.0 33.7 85.4 37.8 0.2 1.32 94.5 6.3 46. Uzbekistan 2000 21.1 205.4 161.4 89.7 31.5 7.0 0.8 2.58 57.1 47. Austria 2003 13.2 94.5 73.5 30.4 5.9 0.3 1.37 48. Belarus 2003 93.2 23.3 73.1 37.0 12.2 2.2 0.1 1.21

Appendix Table A.2 Age Specific Fertility Rates (ASFR) for 88 Countries around the World for the Years 1999-2003

Appendix Table A.2 (Continued)

	40.4	80.5	75.3	37.2	11.3	1.8	0.1	1.23
49. Bulgaria 2003 50. Croatia 2003	40.4 14.0	66.9	93.1	64.2	23.0	4.2	0.1	1.23
51. Czech 2003	14.0	53.8	93.1 94.4	57.4	17.6	4.2 3.1	0.2	1.19
52. Denmark 2003	6.0	46.4	94.4 125.6	121.2	46.6	7.7	3	1.78
53. Estonia 2002	21.9	40.4 76.4	88.6	58.0	24.3	4.9	0.1	1.78
54. Finland 2003	10.4	70.4 57.0	115.5	106.9	24.3 49.4	4.9	0.1	1.75
55. France 2002	8.1	57.0 55.9	129.4	117.7	49.4 51.7	10.8	0.5	1.75
	0.1 11.7	50.9 50.5	86.3	78.4	33.5	5.5	0.0	1.33
56. Germany 2003 57. Gibraltar 2001	24.9	50.5 77.5	168.3	110.8	43.4	5.5 7.9	0.2	2.16
58. Greece 2003	24.9 11.1	43.7	82.3	77.7	43.4 34.9	6.3	0.0	1.28
	20.9	43.7 56.4	88.8	62.0	23.3	4.0	0.7	1.28
59. Hungary 2003	20.9 16.2	50.4 75.9	130.3	02.0 115.7	23.3 48.5	4.0 11.8	0.1	1.20
60. Iceland 2003	16.2 18.8							
61. Ireland 2003	7.1	50.9 35.7	92.5 78.1	133.4 84.6	81.2 41.9	15.5 7.7	0.4 0.4	1.96 1.28
62. Italy 2003		35.7 72.6	78.1 80.3	84.6 51.2	41.9 21.1			1.28
63. Latvia 2002	16.0					4.9	0.4	
64. Liechtenstein 2003	8.8	32.2	91.9 95.2	90.1	40.0	5.9	0.8	1.35
65. Lithuania 2003	20.5	76.0	85.2	46.4	19.1	4.2	0.2	1.26
66. Luxembourg 2003	11.1	57.5	103.4	102.5	43.9	8.3	0.4	1.64
67. Malta 2003	15.9	53.4	101.1	81.6	29.1	5.2	0.1	1.43
68. Netherlands 2003	7.1	41.3	109.6	131.3	52.9	7.0	0.3	1.75
69. Norway 2002	10.1	59.5	121.0	109.3	44.1	7.7	0.2	1.76
70. Poland 2003	14.5	64.1	88.1	52.9	20.9	4.6	0.2	1.23
71. Portugal 2003	20.1	51.2	89.7	84.6	35.7	7.1	0.4	1.44
72. Repub. Moldova 2003	29.2	89.7	67.6	40.7	13.8	2.6	0.1	1.22
73. Romania 2003	34.0	79.7	79.0	41.7	16.1	3.1	0.2	1.27
74. Russian Federation 2001	27.6	94.6	70.6	39.0	13.4	2.4	0.1	1.24
75. San Marino 2003	8.0	22.2	66.4	91.5	50.0	9.0	1.0	1.24
76. Serbia 2001	25.3	105.7	113.6	66.3	25.9	5.1	0.4	1.71
77. Slovakia 2002	21.5	68.6	83.4	46.0	16.2	2.9	0.1	1.19
78. Slovenia 2003	5.8	44.3	94.8	70.7	21.8	3.5	0.1	1.20
79. Spain 2001	10.0	27.4	65.5	95.1	45.8	7.4	0.4	1.26
80. Sweden 2002	6.6	47.7	109.2	110.7	47.3	8.9	0.3	1.65
81. Switzerland 2002	5.4	40.7	89.9	94.5	40.4	6.8	0.4	1.39
82. Yugoslav 2003	25.4	112.1	127.9	64.4	19.7	3.4	0.2	1.77
83. Ukraine 2003	28.6	93.6	67.3	33.1	10.7	2.0	0.1	1.18
84. U. K. 2003	26.6	70.1	95.9	94.5	45.9	9.1	0.5	1.71
85. Australia 2003	16.1	53.8	102.8	112.7	54.4	10.0	0.5	1.75
86. New Caledonia 2003	20.3	109.2	129.5	118.9	64.8	18.3	0.3	2.31
87. New Zealand 2003	26.2	68.9	110.2	114.5	59.3	12.2	0.6	1.96
88. Tonga 1999	28.3	128.3	220.2	201.6	128.2	49.3	3.2	3.80

Source :- United Nations (2006) U.N. Demographic Year Book 2003, Regular Issue, Statistics Division, United Nations, New York.