Migration-Fertility Trade-Off in Low Fertility Populations

Edward Jow Ching TU
Division of Social Science
Hong Kong University of Science and Technology
Kowloon, Hong Kong
soejctu@ust.hk

Xin YUAN
Institute of Population and Development
Nankai University
Tianjin, China
yuanxin@nankai.edu.cn

Long Abstract

Research questions: To what extent increasing migration could alter the trends of the aging and forecasted decline of the populations?

Assumption: (the positive or negative) net-migration is proportional to births; Stable Population with Net-Migration Proportional to Births

Births and Migration:
Density of births at time t: b(t) = beρt, where ρ: the (positive or negative) growth rate
b > 0: the density at t = 0.
p(x) : the probability of surviving to age $0 < x \leqslant \omega$; ω is the highest possible age.
Suppose:
T > 0: the total fertility rate,
α > 0: the lowest age of childbearing,
β is the highest age of childbearing with $\alpha < \beta < \omega$.
The age-specific fertility in age $\alpha \leq x \leq \beta$ is of the form Tf(x), where $f(x) \geq 0$ integrates to 1 over childbearing ages.
Assuming the same survival and fertility values apply to all members of the population of interest, as long as they stay in it.

R(x,t) : the cumulative in-migration to the population of interest, by members of the cohort born at t, by age $0 < x \leq \omega$.
S(x,t) : the cumulative out-migration from the population of interest, by those born at t, by age $0 < x \leq \omega$.
Their densities are $(d/dx)R(x,t) = r(x,t)$ and $(d/dx)S(x,t) = s(x,t)$.
$R(\omega,t) = R(t)$ and $S(\omega,t) = S(t)$, for short.

The age-patterns of the migration streams do not change over time, so $r(x,t) = r(x)R(t)$ and $s(x,t) = s(x)S(t)$, where $r(x) \geq 0$, $s(x) \geq 0$, and (1)

$$\int_0^\omega r(x)\,dx = 1, \quad \int_0^\omega s(x)\,dx = 1.$$

N(x,t) : net-migration to the cohort born at time t, so $n(x,t) = r(x)R(t) - s(x)S(t)$ is its (positive or negative) density in age $0 < x \leq \omega 2$
Define $N(t) = R(t) - S(t)$.

Migration Proportional to Births:
The *proportionality assumption* that leads to stability is: $R(t) = cRb(t)$ and $S(t) = cSb(t)$ for some $cR, cS \geq 0$.
$n(x,t) = h(x)b(t)$, where $h(x) = cRr(x) - cSs(x)$.
We define a function (2)

$$H(x) = \int_0^x h(y)\frac{p(x)}{p(y)}dy.$$

Allowing net migration, so $b(t)H(x)$: the contribution of net-migration to the density of population in age x.
Assuming that survival is independent of the propensity to leave.
Using (2), the density of the population in age x at t as

$$b(t - x)(p(x) + H(x)).$$

Population Renewal:
Earlier births generate the births at t via the basic renewal equation (3)

$$b(t) = T \int_{\alpha}^{\beta} b(t-x)(p(x) + H(x)) f(x)\,dx.$$

Assuming data on net-migration only.
Assuming cR - cS = c $\neq$ *0.*

First, n(x) = h(x)/c, so n(x) integrates to 1.
Second, G(x) = H(x)/c.
This is the same as a normalized version of (2), when h(y) is replaced by n(y). Thus, both n(x) and G(x) are independent of the level N(t) of net-migration. Only the age-pattern matters.
G(x): a *migration survivor function* -- positive or negative "fraction" of the cumulative net-migration surviving to age $0 < x < \omega$. Now H(x) = cG(x) in (3).

The survival probability p(x) and the migration survivor function G(x) are generally fixed so v(x,c) = p(x) + cG(x), for short. Substituting the exponential form of the births into (3) we get the equation (4)

$$1 = T \int_{\alpha}^{\beta} e^{-\rho x} v(x,c) f(x)\,dx.$$

This connects the three parameters D, c, and T.

In analogy with the closed population, the stable population experiencing proportional net-migration has an age-distribution proportional to e -ρx v(x,c).

Effect of Migration on Growth Rate:
In a multi-state system the proportionality factor c is determined as a part of the stable population calculation. We use the empirically observed ratio of current net-migration and current births.

Trade-Off between Fertility and Migration:
For any value of D, consider (4) as defining a relation between T and c. Since we can solve for T > 0 in terms of c, and for c in terms of T, the relation is one-to-one. Taking D = 0, a special case of a stationary population.

Aging via Dependency Ratios:

Dependency Ratios

In a closed stable population, age-distribution is proportional to e $-\rho x$ p(x) for $0 \le x \le \omega$, so a growing population with $\rho > 0$ is older than a declining population with $\rho < 0$.

However, if we fix ρ in an open population, there is a trade-off between c and T. Thus, there can potentially be a second aging effect that derives from the relative values of c and T.

Age-dependency ratios are most often motivated by economic considerations. However, as we concentrate on reproduction, we will first define (5)

$$I_1(c,\rho) = \int_0^{\alpha} e^{-\rho x} v(x,c)\,dx, \quad I_2(c,\rho) = \int_{\alpha}^{\beta} e^{-\rho x} v(x,c)\,dx, \quad I_3(c,\rho) = \int_{\beta}^{\omega} e^{-\rho x} v(x,c)\,dx.$$

The lower dependency ratio L(c,ρ) = I1(c,ρ)/I2(c, ρ), the upper dependency ratio U(c,ρ) = I3(c,ρ)/I2(c,ρ), and the overall dependency ratio D(c,ρ) = L(c,ρ) + U(c,ρ).

Using these measures, net-migration induces "aging", if L(c,ρ) decreases or U(c,ρ) increases with c.