Tempo effect in first marriage table: Japan and China

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Introduction

Recently many East Asian countries and regions are undergoing the rapid increase in age at marriage. The postponement of or retreat from marriage has been the predominant factor of fertility decline in these regions. Researchers there are increasingly aware of the need of most precise measurement of marriage decline. It is well-known that for the measurement of most recent situation of marriage, the life-time proportion ever-married according to the first marriage table is more preferable to the total first marriage rate since the former is based on the intensity of first marriage at the time of observation without the influence of past marriages. However, as Bongaarts and Feeney (1998) argued, the quantum of vital events calculated by life table at certain time is also influenced by tempo change of the occurrence of the events of cohorts. This paper provides, first, a theoretical explanation of the tempo effect in the quantum by the life table in a clearer way than has been previously provided, and an examination of the way to adjust this influence. Second, it provides an application of these theoretical examinations to the first marriage data of Japan and China (Taiwan) and examines the effectiveness and the practicability of the theory.

Theory of tempo effect in the life table

Let $l(x,t)$ denote the proportion of persons who were born at time T= $t - x$ and are surviving without experiencing an vital event, say death, at age x at time t, $d(x,t)$ denote the density of the event of this cohort at age x at time t, and $\mu(x,t)$ denote the corresponding intensity, or force of mortality for death. It should be noted that except $\mu(x,t)$, these functions are different from those for the synthetic cohort hypothesized in the conventional period life table. I call these functions as two-dimensional cohort life table function. These notations are exactly the same as appeared in Bongaarts and Feeney (2003).

I will first clarify the relationships among these functions and define the quantum and the tempo of vital events; and second, define the period life table functions using these cohort life table functions; and third, prove the existence of the tempo effect in the period quantum of the events obtained by the period life table.

At time t that is after x from time T (=t-x), $l(x,t)$ can be expressed by $l(x,t) = l(x, t - x + x)\,|^{t-x=T}$ and $-\mu(x,t)$ also by as follows.

$$-\mu(x,t) = \frac{1}{l(x,t)}\frac{dl(x,t-x+x)|^{t-x=T}}{dx} = \frac{d\log l(x,t-x+x)|^{t-x=T}}{dx} \quad (1.10)$$

Noting of $-\mu(x,t) = -\mu(x,t-x+x)|^{t-x=T}$, integrating the both sides of (1.10) , we obtain

$$[\log l(a,t-x+a)]_0^x = -\int_0^x \mu(a,t-x+a)da \,,$$

$$l(x,t) = \exp\left[-\int_0^x \mu(a,t-x+a)da\right], \quad (1.11)$$

Using the relation of $d(x,t) = l(x,t) \quad \mu(x,t)$, we get the following expression

$$d(x,t) = \mu(x,t)\exp\left[-\int_0^x \mu(a,t-x+a)da\right], \quad (1.13)$$

Using this expression, after multiplying $l(x,t)$ to the both sides of equation (1.10) and integrating them from age 0 to age x, we obtain

$$\int_0^x d(a,t-x+a)da = -[l(a,t-x+a)]_0^x = 1 - l(x,t) \quad (1.14)$$

Hence, $Q_c(T)$ the mean life-time experience frequency per person is given as follows for the cohort born at time $T = t - x$ at age $x = \omega$ at time $t = t - \omega + \omega = T + \omega$.

$$Q_c(T) = \int_0^\omega d(x,T+x)dx = 1 - l(\omega,T+\omega) \quad (1.15)$$

Meanwhile, the period life table based on $\mu(x,t)$ is expressed by the period life table functions as follows. Note that the integration and the differentiation in these functions are not in the direction of the cohort life course line but in the direction of age of the hypothetical cohort at time t.

$$l_p(x,t) = \exp\left[-\int_0^x \mu(a,t)da\right], \quad d_p(x,t) = -\frac{\partial l_p(x,t)}{\partial x} \quad (2.1)$$

Also, $\mu(x,t) = \dfrac{d_p(x,t)}{l_p(x,t)}$

$$d_p(x,t) = \mu(x,t) \exp\left[-\int_0^x \mu(a,t)da\right] \quad (2.2)$$

From the occurrence density $d_p(x,t)$, the quantum Q_p, and mean of events occurrence M_p can be defined as follows, postulating $l_p(0,t) = 1$.

$$Q_p(t) = \int_0^\omega d_p(x,t)dx = \int_0^\omega \mu(x,t)l_p(x,t)dx = \int_0^\omega -\frac{\partial l_p(x,t)}{\partial x}dx = 1 - l_p(\omega,t)\,, \quad (2.3)$$

$$M_p(t) = \frac{\int_0^{\omega} x d_p(x,t)dx}{\int_0^{\omega} d_p(x,t)dx} = \frac{\int_0^{\omega} l_p(x,t)dx - \omega l_p(\omega,t)}{1 - l_p(\omega,t)} \quad (2.4)$$

I call $Q_p(t)$ as the period life table quantum, and $M_p(t)$ as the period life table mean age at event.

Tempo change in cohorts and the influence on period quantum

McKendrick equation for a cohort with size $l(x,t)$ can be expressed as follows;

$$-\mu(x,t) = \frac{1}{l(x,t)}\frac{\partial l(x,t)}{\partial t} + \frac{1}{l(x,t)}\frac{\partial l(x,t)}{\partial x} \quad (1.19)$$

If we express here the growth rate of $l(x,t)$ at age x at time t as $r(x,t) = \frac{1}{l(x,t)}\frac{\partial l(x,t)}{\partial t}$,

then we obtain

$$r(x,t) - \mu 1(x,t) = -\mu(x,t) \quad (3.1)$$

where $\mu 1(x,t) = \frac{d1(x,t)}{l(x,t)} = -\frac{1}{l(x,t)}\frac{\partial l(x,t)}{\partial x}$ and $d1(x,t) = -\frac{\partial l\ (x,t)}{\partial x}$ (2.5)

This relation is well-known for age specific population（Bennett and Horiuchi, 1981; Preston and Coale, 1982; Arthur and Vaupel,1984）.

The Integration of the first equation of (2.5) yields

$$l(x,t) = \exp\left[-\int_0^x \mu 1(a,t)da\right] \quad (2.7)$$

Using this equation, another period quantum can be defined as follows.

$$Q1(t) = \int_0^{\omega} d1(x,t)dx = \int_0^{\omega} \mu 1(x,t) l(x,t)dx = \int_0^{\omega} -\frac{\partial l(x,t)}{\partial x}dx = 1 - l(\omega,t), \quad (2.8)$$

Substituting $-\mu 1(x,t) = -\mu(x,t) - r(x,t)$ in equation (2.7) and using equation (2.1), it reduces to

$$l(x,t) = \exp\left[-\int_0^x \{\mu(a,t) + r(a,t)\}da\right] = l_p(x,t)\exp\left[-\int_0^x r(a,t)da\right] \quad (3.3)$$

Tempo effect in life table functions

Let us examine the tempo effect in the quantum in the period life table $Q_p(t)$ defined by equation (2.3).

Let x=ω in equation (3.3). Then we obtain

$$l(\omega,t) = \quad l_p(\omega,t)\exp\left[-\int_0^{\omega} r(a,t)da\right] \qquad (3.8)$$

Substituting this in equation (2.8), it reduces to

$$Q1(t) \; = 1\text{-}\, l_p(\omega,t)\exp\left[-\int_0^{\omega} r(a,t)da\right] \qquad (3.9)$$

According to equation (2.3), $l_p(\omega,t) = \ 1 - Q_p(t)$, equation (3.9) reduces to

$$Q1(t) \ = \ 1-\{1-Q_p(t)\}\exp\left[-\int_0^{\omega} r(a,t)da\right] \qquad (3.10)$$

This equation represents the relation between a period quantum $Q1(t)$ and the quantum by the life table $Q_p(t)$.

Now we postulate that the proportion of people who have never experienced an event at maximum age ω, $l(\omega,t)$ be constant. That is the postulation that the quantum of events of the cohorts $Q_c(T)$ is constant even if the tempo of the event $M_c(T)$ changes with regard to cohorts.

According to this postulation, equation (1.15) reduces to $Q_c(T)$=1- $l(\omega,t)$. This is the same equation with (2.8), i.e. $Q_c(T)$ = $Q1(t)$ = 1- $l(\omega,t)$. Hence, under this postulation, there is a relation between $Q_c(T)$ and $Q_p(t)$ shown below.

$$Q_c(T) = \ 1-\{1-Q_p(t)\}\exp\left[-\int_0^{\omega} r(a,t)da\right] \qquad (3.11)$$

$$\frac{l_p(\omega,t)}{l(\omega,t)} = \frac{1-Q_p(t)}{1-Q_c(T)} = \ \exp\left[\int_0^{\omega} r(a,t)da\right] \qquad (3.12)$$

Hence, if $\int_0^{\omega} r(a,t)da >$ 0, then appears the following relation.

$$l_p(\omega,t) \ > l(\omega,t), \qquad Q_p(t) \ < Q_c(T) \qquad (3.13)$$

The reverse relation emerges in the reverse case.

These relations show the existence of the distortion in the quantum obtained through period life tables ($Q_p(t)$) in that it will differ from cohort quantum ($Q_c(T)$) when the tempo of cohort life table changes, i.e. $r(x,t)$, growth rate of $l(x,t)$ changes so that $\int_0^{\omega} r(a,t)da \neq 0$.

Adjustment of quantum of period life table

The distorted quantum obtained through the period life table is to be adjusted so that it represents the average level of the quantum of cohorts that constitute the period quantum.

For adjusting the period quantum derived from the intensity of the vital events, some authors proposed the shift model of intensity (Bongaart and Feeney, 2006; Kohler and Ortega, 2002). This model is not applicable, however, if the intensity at the ages higher than the average age hardly changes in spite of the shift of the average age of the intensity. In fact for first marriage, it often is the case. (I will show this for first marriage in Japan and Taiwan in the next section.) If we take the density of the events by the life table, however, the shifting of the age pattern is more salient. Therefore, it is appropriate to apply the shifting model to adjust the distortion in the period quantum of the events $Q_p(t)$ that can be assumed to be derived from period density $d_p(x,t)$ ((2.2)) of the events. The adjustment procedure that is considered to be appropriate is the one proposed by Bongaart and Feeney, 1998 for fertility, i.e. $Q_p(t) \Big/ 1-\dfrac{dM_p(t)}{dt}$,

using the time derivative of the average age of the event $M_p(t)$.

It should be noted, however, that the density of the event is for the period or the synthetic cohort and not for real cohort. Therefore, the consistency among densities by age for a cohort is not strictly maintained. For example, if the event is death, the sum of the densities by age for a cohort does not necessarily coincide with unity. It may be one of the sources of the error of this adjustment. In this sense, the adjustment is an approximation.

Application to first marriage in China (Taiwan) and Japan

First marriage table was calculated by the vital statistics and the population census for every five years from 1980 to 2000 for Japan and for 2000 and 2007 for Taiwan*1. Figure 1 shows that the delay of first marriage depicted by first marriage table is not caused by the simple shift of the age pattern of intensity of first marriage. The intensity at the ages higher than the average age hardly changes while that at lower ages is declining.

Figure 2 shows, however, that the delay of first marriage is caused by the shift of the density of the first marriage $d_p(x,t)$ although the shape is also changing in a way that the top of the curve is going down. It is plausible to assume that the change in the density of first marriage can be the shift at least for a short period of time. Therefore, I adjusted the period quantum via the first marriage table by the formula above-mentioned.

The results are presented in Table 1 for Japan and in Table 2 for Taiwan.

In both regions, the proportion ever-married by first marriage table has been getting smaller for both sexes. The adjusted proportion ever-married becomes higher by about 5 percent than the counterpart. Adjusted values surpassing unity for females in Japan may be caused by the adjusting formula I adopted and by data problem in Taiwan cases.

The proportion of ever-married among Japanese females in 2000 by the first marriage table is 0.83 and the adjusted value is 0.89. These values could be the maximum level of marriage when we project the future trend in women's marriage in Japan. The converging value of the proportion of ever-married women is set as 0.764 in the official population projection conducted by the Japanese government in 2006, which is very low, compared with these values.

Note

1) First marriage tables were provided by Motomi Beppu. I appreciate his cooperation.

2)

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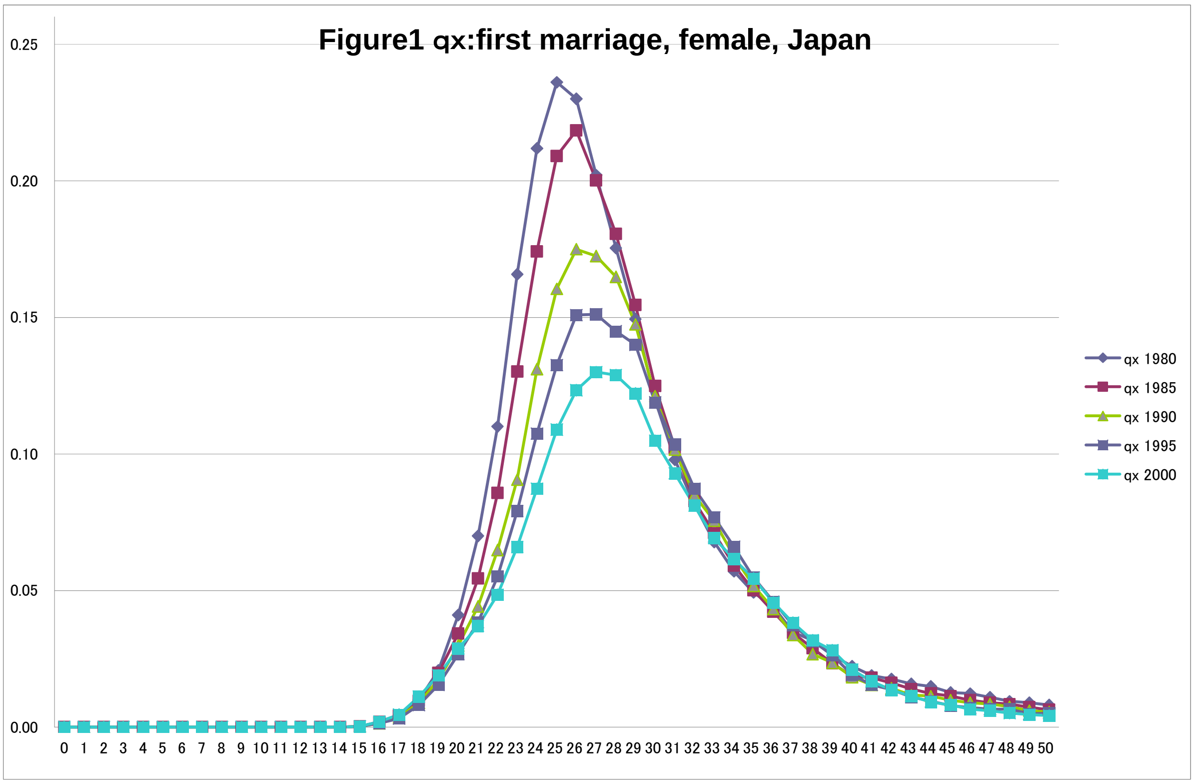
Figure1 qx:first marriage, female, Japan
0.25
0.20
0.15
0.10
0.05
0.00
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50
qx 1980
qx 1985
qx 1990
qx 1995
qx 2000

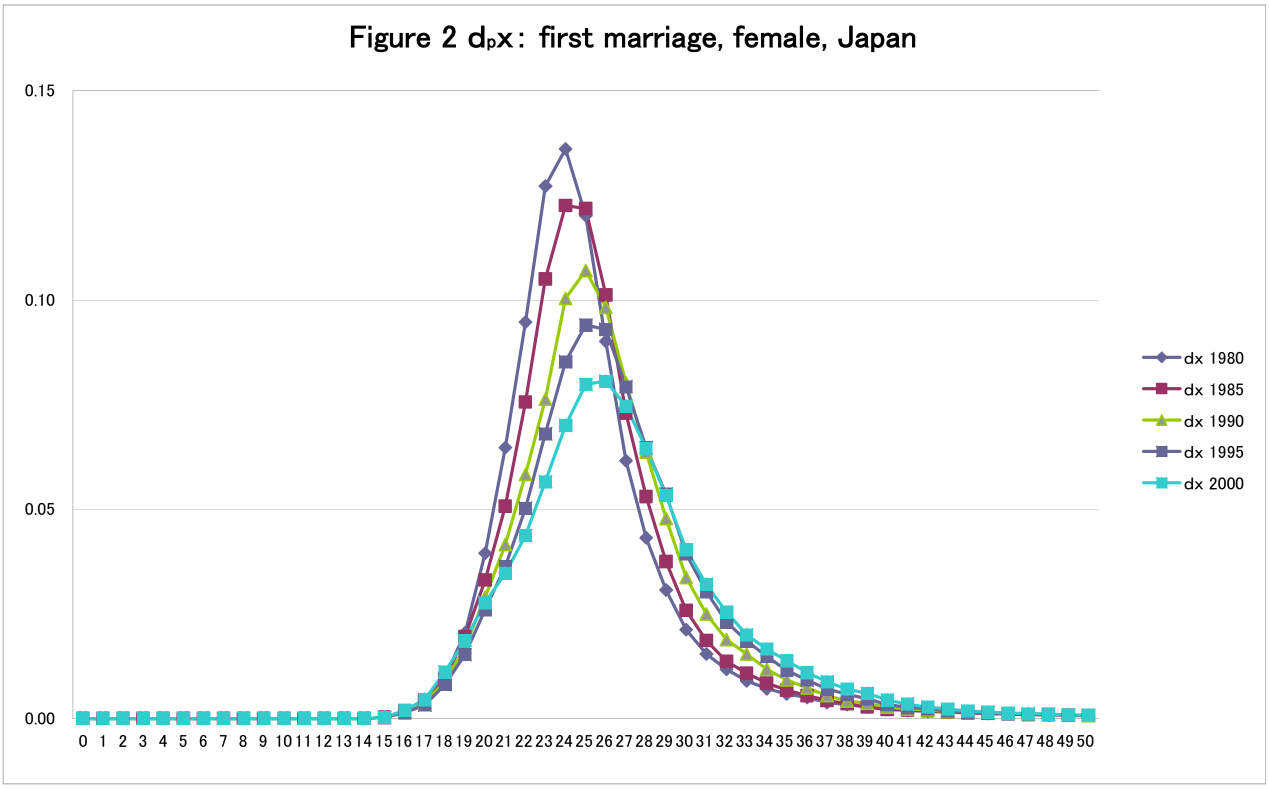
Figure 2 $d_p x$: first marriage, female, Japan
0.15
0.10
0.05
0.00
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50
dx 1980
dx 1985
dx 1990
dx 1995
dx 2000

Table 1 Life-time proportion ever-married and mean age at first marriage by first marriage table: Japan

	Male			Female		
Year	Life-time proportion ever-married	Mean age at first marriage (yrs)	Adjusted life-time proportion ever-married	Life-time proportion ever-married	Mean age at first marriage (yrs)	Adjusted life-time proportion ever-married
1980	0.892	28.3		0.950	25.3	
1985	0.858	28.5	0.905	0.928	25.8	1.048
1990	0.832	28.8	0.884	0.890	26.5	1.013
1995	0.819	29.0	0.863	0.867	27.0	0.947
2000	0.778	29.4	0.830	0.832	27.3	0.892

Change rate of age at marriage for 2000 is based on the rate of 1995-2000.

Table 2 Life-time proportion ever-married and mean age at first marriage by first marriage table: Taiwan

	Male			Female		
	Life-time proportion ever-married	Mean age at first marriage (yrs)	Adjusted life-time proportion ever-married	Life-time proportion ever-married	Mean age at first marriage (yrs)	Adjusted life-time proportion ever-married
2000	0.967	32.0	1.193	0.943	28.2	1.298
2007	0.770	33.4	0.950	0.757	30.1	1.043

Change rate of age at marriage is provided based on the rate of 2000-07.

Figure 3 Life-time proportion ever-married and mean age at first marriage by first marriage table: Japan

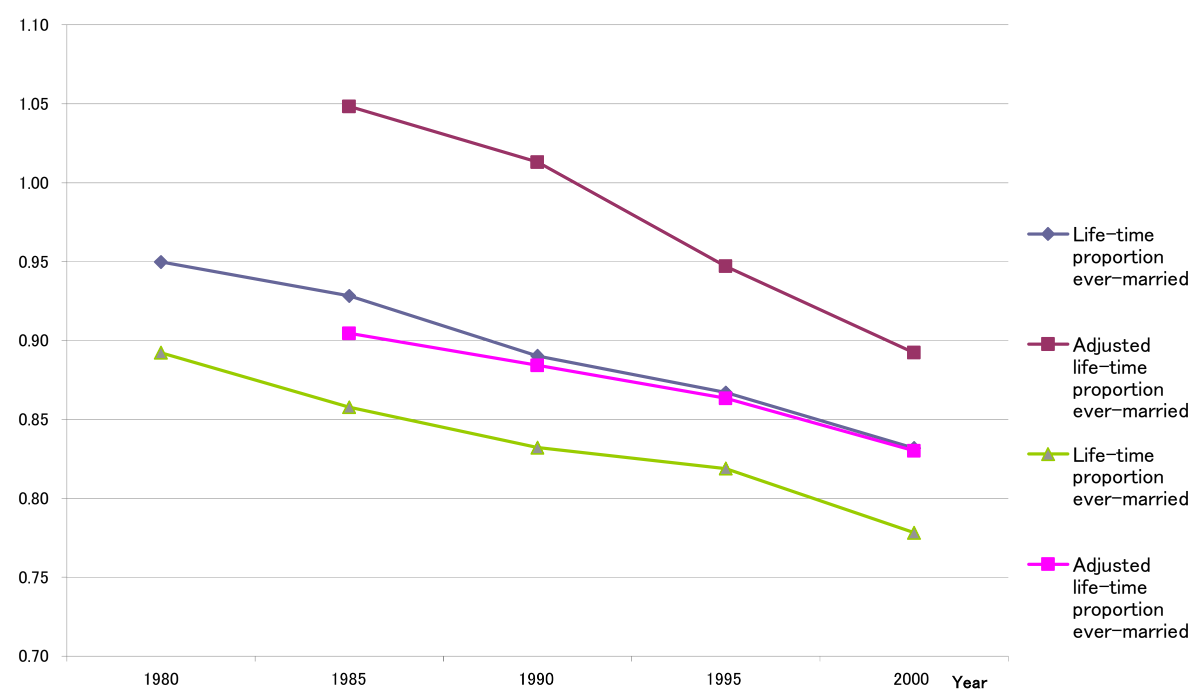

Figure 4 Mean age at first marriage by first marriage table:Japan