

Accounting for Intragenerational Mobility on Poverty and Inequality *

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STUDY OBJECTIVE.

The ability to move along the social scale is a fundamental determinant of one's wellbeing because it affects the dynamics of poverty and inequality. The overall objective of this chapter is to measure net intragenerational mobility between income classes in Brazil using demographic projection methods. Specific aims are twofold: (1) to suggest a method to calculate intragenerational net mobility using the residual difference between projected and observed censal populations. The main advantage of this approach is to estimate net mobility by age using only cross-sectional data by income class; (2) to estimate how income-specific net mobility has changed between 1980 and 2000 in Brazil in the presence of concurrent fertility and mortality declines. More specifically, this chapter will demonstrate the importance of social mobility vis-à-vis demographic changes in the determination of poverty. I identify the ages and income classes at which net mobility is positive, or negative, and describe the age standard of mobility and mortality in two periods, between 1980 and 1990, and between 1990 and 2000.

BACKGROUND AND SIGNIFICANCE.

The motivation for this study lies in a deceptively simple question: Why has income inequality changed relatively little in the face of one the most rapid demographic transitions ever experienced in Brazil? More specifically, why the rapid reproductive convergence of the poorest

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and richest income classes did not seem to have affected inequality more markedly? Previous studies have offered a simple answer: because there is social mobility between groups homogenizing the size of income classes (e.g. Lam 1986; Mare 1997; Preston and Campbell 1993). When upward social mobility is high, one can be born into a poor family and still break the poverty trap by moving upwards. That is, social mobility may counterbalance the trends implied by demographic differentials. Mare (1997), for instance, showed that differential fertility by educational attainment have had small effects on educational trends, partly because of intergenerational mobility and partly because differential fertility have not been large enough to have a large effect in the long term.

Any attempt to measure the dynamics of poverty and inequality must take into account how income groups reproduce and also how members of these groups move between them. When ascendant mobility for those at the bottom of the income distribution is higher than for those at the top, the class or origin is less important in determining income inequality and poverty status than in situations where mobility is low.

In the United States there are several studies in Sociology and Economics dealing with *intergenerational* mobility. Comprehensive reviews of what has happened to intergenerational mobility in the last century in the United States are available (McMurrer, Condon and Sawhill 1997; Morgan, Grusky and Fields 2006). In Latin America, good comparative studies have also been produced (Behrman, Gaviria and Székely 2001). In Brazil, the mobility literature has focused attention on education (Barros and Lam 1993; Ferreira and Veloso 2003), occupation (Pastore 1982; Pastore and Silva 2000), income (Ferreira and Veloso 2004; Pero and Szerman 2005) and on the mobility of elites (Ferreira 2001; Medeiros 2005). Yet studies dealing with

intragenerational mobility are virtually inexistent in Brazil, largely due to the lack of longitudinal data.

Research on intragenerational mobility was more prevalent in American Sociology during the 1970s and 1980s (DiPrete 1993; Rosenfeld 1992; Spilerman 1977), but more recent research emphasizing career processes is also available (DiPrete and Nonnemaker 1997; Spilerman and Petersen 1999). Attention to the “life cycle approach” made intragenerational mobility important on its own right, although the topic is “rarely studied as such anymore” (Morgan et al. 2006: 7). Intragenerational mobility is an important research topic in Sociology when the goal is to understand one’s status achievement over the life cycle rather than in relation to past generations. Since the ultimate goal is to unveil the dynamic evolution of wellbeing through time, it should matter little if the reference group is the parents’ generation or early periods of one’s own life cycle. Because intragenerational mobility presents the experience or real cohorts, it enables the location in time of acts of mobility linked to structural characteristics of society (Sørensen 1975: 457). From an operational point of view, one comparative advantage of examining mobility from an intragenerational perspective is that longitudinal data, rarely available in developing countries, is not required since the group of analysis refers to age cohorts rather than individuals. Moreover, in intragenerational cohort studies information about the parents and retrospective data is not required, so recall errors are minimized.

Standard procedures to measure inter and intragenerational mobility usually require longitudinal or retrospective questions about states occupied in the present and in the past. Once current and past positions – in socioeconomic status, income, region, race, marital status and others – are compared, one can infer the likelihood of moving upwards, downwards, or of remaining in the same category of origin. The absence of retrospective questions in household

surveys have, however, hindered research on mobility in less developed countries, where the cost of implementing follow up surveys is prohibitive.

In this chapter I suggest an innovative way of measuring income net mobility¹ combining cross-sectional data and demographic projection methods. The method estimates net intragenerational cohort mobility by age and between three income classes: poor, middle and rich. One advantage of this approach is that it takes into account the role of demographic differentials between income groups (e.g. differences in fertility and mortality) to offer a picture of net mobility over the life cycle. It also provides an initial strategy to examine the “selection hypothesis”, which suggests that upward mobility is impeded by large family sizes and the probability of downward mobility exacerbated (Blau and Duncan 1967; Van Bavel 2006). This hypothesis holds when large families have to spend resources (time, money, effort) with children and may therefore find it difficult to maintain or improve their social position. Couples with low fertility may also find it easier to move upwards in the social scale because they can use their extra resources to improve their chances for status achievement.

Although the bulk of previous research has treated social mobility as “independent” from fertility, there is reason to suspect that fertility affects mobility, or that there is some degree of mutual reciprocity between the two variables. (Kasarda and Billy 1985) provide an excellent review of the theoretical and empirical causal links between social mobility and fertility. In this chapter I take net mobility as a residual outcome of the difference between recorded and projected populations by income class and age. Mobility is, therefore, an indirect product of fertility and mortality because these two demographic components are what define the projection. The next section elaborates on the methodological details.

¹ For the sake of simplicity, from now on the term mobility refers to intragenerational cohort mobility. Cohorts are defined by age, but not by sex, for three different income groups: poor, middle and rich classes.

DATA AND METHODS.

The dataset includes special tabulations of family per capita income by age, available in the 1980, 1991 and 2000 Brazilian Censuses produced by the *Instituto Brasileiro de Geografia e Estatística* (IBGE). The Brazilian censuses are publicly available at IPUMS International website (Ruggles et al. 2004).

Measuring income mobility.

A general definition states that income mobility “measures how individuals or families move within the income distribution over time” (Gottschalk and Dazinger 1998: 20). This definition is perhaps too vague to be incorrect, but is certainly incomplete because it does not elaborate on how to measure mobility. To assure consistency, one has to answer: Mobility between what, when and how? To answer the “what”, I reiterate that this chapter deals with *intragenerational* mobility. It refers to mobility within the same generation and between income classes, but not necessarily for the same individuals. Instead of tracking individuals or households, I track entire cohorts of individuals based on their age and income. I refer to mobility as net mobility because there is no information to determine the origin and destiny of moves, but I can infer the net total result of incoming or outgoing individuals in each age group by comparing projected and observed populations in each age and income class. The main advantage of this approach is that it only requires cross-sectional data, so the time dimension of mobility can be extended. In addition, using cohorts instead of individuals tends to dilute measurement errors and reduce attrition. The drawback of this approach, however, is that by looking at average cohort income eliminates the possibility of studying intra-cohort income mobility (Fields et al. 2006).

To answer the “when” of mobility, I reiterate that the period of investigation covers income net mobility in Brazil between three years: 1980, 1991 and 2000. The first period, 1980-1990, is marked by expressive educational expansion (Ferreira and Barros 1999; Wajmann and Menezes-Filho 2003) and by accelerated rate of inflation. The next interval, 1991-2000, is characterized by a slight decline in racial inequality, and by regional convergence of income between states and rural-urban areas (Ferreira et al. 2006). The added value of having three points in time is that *changes* in net mobility can also be observed. As mobility can only be estimated with two or more years of data, at a very minimum, it takes at least three data points to measure changes in mobility. Few studies have looked at changes in *intergenerational* income mobility in the U.S. (Gottschalk 1997; Gottschalk and Dazinger 1998) and only one has looked at changes in Brazil (Ferreira and Veloso 2004). I did not find research examining changes in *intragenerational* income mobility over time.

Finally, to answer “how” to measure mobility I start by defining three income classes between which mobility occurs: poor, middle and rich. Income thresholds separating these three subpopulations are calculated following (Medeiros 2005)². These three classes have fixed income boundaries over time and follow the distribution of family income per capita of 1980. The family per capita income value separating the poor from the middle class in 1980 was equal to R\$80.42 (about \$44.5 Dollars on October 2007), which corresponds to roughly 33 percent of the population at the bottom of the distribution. This absolute poverty threshold remains the same in 1990 and 2000. The rich group accounts for about one percent of those at the top of the income distribution, and the middle class is represented by the subpopulation in between the rich and the poor.

² See the methodological section of previous chapters for further details on the calculation of income classes, inequality measures and estimation of income-specific demographic rates.

The strategy to estimate mobility between these three income classes consists in comparing the difference between observed and projected populations. The residual difference in the size of projected and recorded populations in each income class is attributed to international migration and net social mobility.³ The logic of the method consists in acknowledging two forces of decrement in each age: the first is mortality, which is estimated using indirect demographic methods described in the previous chapter⁴; the second force of decrement (or increment) is net social mobility, which can be estimated as the complement of mortality needed to match projected and recorded censal populations. This methodological strategy follows the logic of multiple decrement/increment processes to combine forces of mortality and mobility into a single framework expressed in Leslie matrices, which can then be used to project and generate counterfactual scenarios of population growth. A detailed explanation of how to estimate the influence of intragenerational mobility is in the next section.

The matrix model of interregional cohort-survival with three subpopulations.

Building on the matrix framework of interregional population growth and distribution developed by Rogers (1968, 1975), this section demonstrates how information on intragenerational mobility can be retrieved by comparing projected and observed subpopulations. The method has two assumptions. The first is that mortality and net social mobility can be expressed as part of a multiple decrement process for each subpopulation. The second assumption is that mortality and net mobility are constant during the transition periods. Therefore, the chance of dying or moving

³ Preliminary results show that international migration flows do not make much difference and that most of the difference between projected and observed populations is due to social mobility.

⁴ A copy of the referred chapter is available under request to the author.

between states will be the same for each period of analysis (e.g. for each projection period of five years).

For a single population, the general model surviving an age distribution forward through time may be expressed by a summation of matrices multiplication according to equation (1) defined as:

$$\mathbf{w}^{t+1} = \mathbf{S}\mathbf{w}^t + \mathbf{M}\mathbf{w}^t \quad (1)$$

where,

$$\mathbf{S}_{n \times n} = \begin{bmatrix} 0 & b_1 & b_2 & \cdots & b_u & 0 & 0 \\ d_{r+1} & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & d_{r+1} & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & d_{r+2} & \cdots & \cdots & \cdots & 0 \\ \vdots & & & \ddots & & & 0 \\ \vdots & & & & \ddots & & \\ 0 & & & & & d_{n-1} & 0 \end{bmatrix}, \quad \mathbf{w}^t = \begin{bmatrix} w_r^t \\ w_{r+1}^t \\ w_{r+2}^t \\ \vdots \\ \vdots \\ w_{n-1}^t \\ w_n^t \end{bmatrix}, \quad \mathbf{M}_{n \times n} = \begin{bmatrix} 0 & 0 & \cdots & \cdots & 0 \\ m_1 & 0 & & & 0 \\ 0 & m_2 & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & \cdots & m_{n-1} & 0 \end{bmatrix}$$

w_r^t = population in the r th age group at time t ;

b_r = number of births surviving to the $t+1$ in the r th childbearing age group;

d_r = proportion of people who “survived” between ages r and $r+1$ st between t and $t+1$.

m_i = net migration/mobility rate for the i th age group.

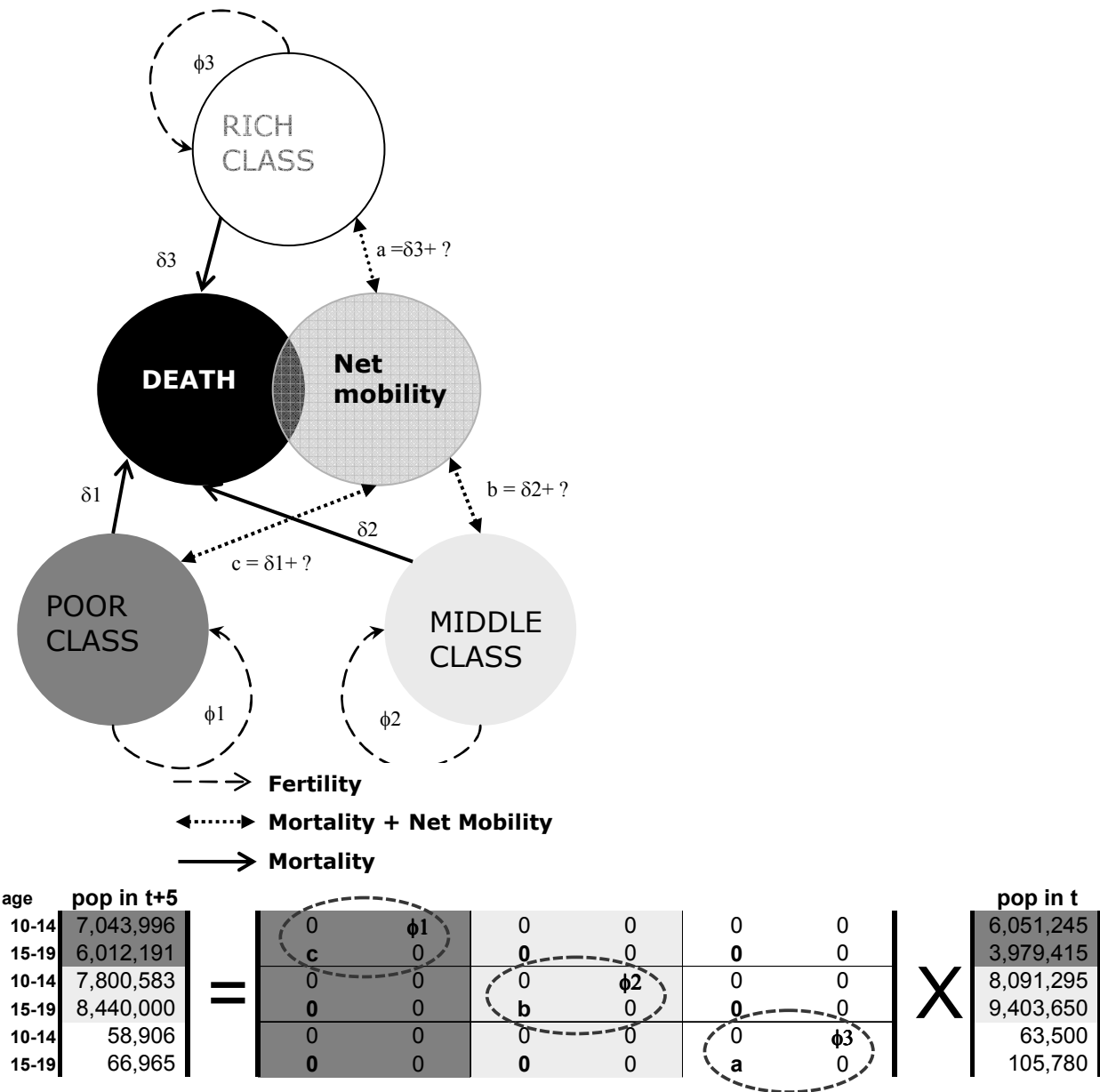
The elements in the subdiagonals of \mathbf{S}_1 and \mathbf{S}_2 account for the “mortality” of those individuals who left age group r to $r+1$ st through death. The state-age specific elements in matrices \mathbf{S}_i can be fully retrieved making use of standard demographic techniques. Fertility and survivorship rates can be obtained with direct and indirect demographic methods (e.g. Brass et al. 1968) discussed in the previous chapter. The elements in matrices \mathbf{M}_{ij} , which describe the net

proportion of people in age group r th who made the transition from state j into state i and into the next age group, are usually harder to obtain without mobility information. However, even when detailed information on movements between states is not available, age-specific transition matrices can still be estimated using the logic of multiple decrement processes. Disaggregating the total population into three subpopulations, $\mathbf{w}_{\text{Total}}^t = \mathbf{w}_{\text{Poor}}^t + \mathbf{w}_{\text{Middle}}^t + \mathbf{w}_{\text{Rich}}^t$, we may express the fundamental model of population projection with internal mobility and three groups by combining net mobility and mortality rates. This joint process of population growth can be represented in matrix form as:

$$\begin{bmatrix} \mathbf{w}_{\text{POOR}}^{t+5} \\ \mathbf{w}_{\text{MIDDLE}}^{t+5} \\ \mathbf{w}_{\text{RICH}}^{t+5} \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{\text{POOR}} + \mathbf{M}_{\text{POOR}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_{\text{MIDDLE}} + \mathbf{M}_{\text{MIDDLE}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{S}_{\text{RICH}} + \mathbf{M}_{\text{RICH}} \end{bmatrix} \times \begin{bmatrix} \mathbf{w}_{\text{POOR}}^t \\ \mathbf{w}_{\text{MIDDLE}}^t \\ \mathbf{w}_{\text{RICH}}^t \end{bmatrix} \quad (2)$$

This expression implies that the future population size of each income class can be expressed as a product of the baseline population and a set of fertility and mortality rates (in \mathbf{S}) plus a matrix \mathbf{M} accounting for net migrants from or to other subpopulations. Net mobility rates in \mathbf{M} represent the complement of survival probabilities in \mathbf{S} required to match the size of projected and observed populations. The population projection scheme illustrating population dynamics over time using fertility, survival and net mobility can be diagrammatically expressed as:

Figure 1. Projection scheme of population dynamics with fertility, mortality and net mobility



This multiregional Leslie matrix for two age groups (10-14 and 15-19) and three subpopulations (poor, middle and rich) illustrates the structure required to project the size of poor, middle and rich classes from time t to $t+5$ using a set of fertility (ϕ_1, ϕ_2, ϕ_3) and survival probabilities that include the net mobility of specific classes (parameters a, b, c in the

subdiagonal]. Projected and recorded populations at time $t+5$ have the same size because mobility between income classes has been incorporated into the projection matrices.

The combined “survival probability”, expressed by the sum of matrices **S** and **M**, reflects the joint effect of mortality and net mobility and are expressed as a single parameter for each age and class. Mortality and net mobility are expressed in a single matrix because when projected subpopulations (w^{t+5*}) are larger than observed ones (w^{t+5}), matrix **M** will have negative elements (e.g. emigration from that social class). This is an undesirable result because it compromises the use of **M** as an independent projection matrix since negative entries imply negative organisms. To avoid this outcome I work with only two projection matrices. The first is represented by **S** and includes probabilities of surviving and fertility rates, as in the last chapter. It represents the demographic forces behind the reproduction of income-specific groups. The second multiregional Leslie matrix includes the reproductive forces explicit in **S** but also incorporates net social mobility, represented by the elements of **M**. This second matrix is expressed as $\mathbf{R} = \mathbf{S} + \mathbf{M}$. This combined matrix **R** adjusts the survivorship ratios by incorporating all negative elements of **M** into the proportion of people surviving. This procedure provides a projection matrix with only nonnegative values from which a single dominant eigenvalue and eigenvectors can then be obtained to describe population dynamics in the steady state⁵. Some of the entries in **R** are higher than one because of immigrants coming from other social classes and regions, but that does not compromise the usefulness of the matrix as an analytical scheme.

To illustrate this theoretical discussion I present an empirical example for the projection of the poor population between 1985 and 1990 using Brazilian Census data. The parameters in matrix **S_{POOR}** below reflect the reproduction of the poor population, between ages 0 and 80:

⁵ The concept and use of eigenvalues and eigenvectors in demography is discussed somewhere else (Caswell 2001; Keyfitz and Caswell 2005).

Figure 2. Leslie matrix of the poor class for the period 1985-1990

| age | $w^{t+10} \text{ POOR}^*$ Poor ¹⁹⁹⁰ | S_{POOR} | | | | | | | | | | | | | | $w^{t+5} \text{ POOR}$ Poor ¹⁹⁸⁵ | | | | |
|-----|---|-------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|--|------|------|---|-----------|
| 0 | 8,446,845 | 0 | 0 | 0.12 | 0.41 | 0.60 | 0.54 | 0.39 | 0.24 | 0.09 | 0.02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,760,538 |
| 5 | 7,667,290 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,396,204 |
| 10 | 7,363,506 | 0 | 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,043,996 |
| 15 | 6,998,534 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6,012,191 |
| 20 | 5,947,210 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,936,405 |
| 25 | 3,880,457 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,362,833 |
| 30 | 2,322,412 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,133,777 |
| 35 | 2,088,392 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,101,456 |
| 40 | 2,044,574 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.97 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1,924,381 |
| 45 | 1,856,319 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.96 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1,618,401 |
| 50 | 1,542,824 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.95 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1,119,114 |
| 55 | 1,050,044 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.94 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 873,685 |
| 60 | 801,532 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.92 | 0 | 0 | 0 | 0 | 0 | 0 | 630,254 |
| 65 | 559,718 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.89 | 0 | 0 | 0 | 0 | 0 | 483,437 |
| 70 | 404,832 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.84 | 0 | 0 | 0 | 0 | 420,402 |
| 75 | 319,524 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.76 | 0 | 0 | 0 | 252,756 |
| 80+ | 168,063 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.43 | 0.43 | 0 | 141,623 |

The leftmost column in Figure 2 represents the projected population in 1990, and the right column is the number of people who were poor in 1985 by five year age groups. The first row of the matrix has net fertility rates properly adjusted for the effect of mortality on births. More precisely, the values in the first row reflect the contribution of persons in the r th age group at initial time $t+5$ to the number of persons in the first age group at the end of the interval, time $t+10$ in this case. Survivorship ratios are in the subdiagonal of S_{POOR} and reflect the proportion of persons in the r th age group at initial time $t+5$ who survive to be in the $(i+5)$ th age group at time $t+10$ (Schoen 2006: 12). The projected poor population, however, is different from the one actually registered in 1990, because matrix S does not incorporate net mobility and the impact of the migrants' fertility on the projection. A matrix incorporating these two factors is represented by R_{POOR} :

Figure 3. Leslie matrix of the poor class for the period 1985-1990 incorporating net mobility

| age | w_{POOR}^{t+10} Poor ¹⁹⁹⁰ | $R_{\text{POOR}} = S_{\text{POOR}} + M_{\text{POOR}}$ | | | | | | | | | | | | | | w_{POOR}^{t+5} Poor ¹⁹⁸⁵ | | | | |
|-----|--|---|------|------|------|------|------|------|------|------|------|------|------|------|------|---|------|------|---|-----------|
| 0 | 8,796,586 | 0 | 0 | 0.10 | 0.40 | 0.67 | 0.65 | 0.43 | 0.26 | 0.10 | 0.02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,288,408 |
| 5 | 8,804,529 | 1.06 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,952,320 |
| 10 | 8,190,257 | 0 | 1.03 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,286,970 |
| 15 | 5,901,452 | 0 | 0 | 0.81 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4,900,683 |
| 20 | 4,250,221 | 0 | 0 | 0 | 0.87 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,451,232 |
| 25 | 3,820,620 | 0 | 0 | 0 | 0 | 1.11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,653,442 |
| 30 | 3,468,164 | 0 | 0 | 0 | 0 | 0 | 1.31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,837,480 |
| 35 | 2,981,957 | 0 | 0 | 0 | 0 | 0 | 0 | 1.05 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,256,451 |
| 40 | 2,446,579 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.08 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,144,579 |
| 45 | 1,872,958 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.87 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1,465,247 |
| 50 | 1,511,877 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.03 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1,211,295 |
| 55 | 1,189,421 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 914,340 |
| 60 | 1,005,328 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.10 | 0 | 0 | 0 | 0 | 0 | 0 | 755,354 |
| 65 | 815,921 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.08 | 0 | 0 | 0 | 0 | 0 | 588,009 |
| 70 | 560,997 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.95 | 0 | 0 | 0 | 0 | 478,968 |
| 75 | 381,727 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.80 | 0 | 0 | 0 | 265,039 |
| 80+ | 327,696 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.67 | 0.67 | 0 | 223,101 |

Matrix R_{POOR} above generates a population (w_{POOR}^{t+10}) with the same size as the one in 1990. Age specific fertility rates in R_{POOR} are on average 6.26 percent higher than in S_{POOR} . Some entries in the subdiagonal of matrix R are higher than one to reflect the entrance of people to the poor class at those ages. In the next section I provide a detailed description of how to calculate the survivorship ratios in the subdiagonal of any matrix R ⁶.

Procedure to estimate the joint effect of mortality and net mobility.

For the sake of comparison, the entries in the subdiagonal of matrix R are calculated using two procedures. The first method is based on the variable r-method of intercensal survival described by Preston, Heuveline and Guillot (2001: 184-190). The second procedure is also based on intercensal survivorship ratios but its results are more accurate than the r-method. I use the distribution of the poor populations in 1980 and in 1990 to illustrate these methods below.

⁶ S and R matrices for other income classes are reproduced in Appendix A.

The logic of the r-method consists in assuming that the mean growth during the intercensal period is constant by age. With this assumption, the number of person-years in the life table can be estimated as:

$${}_nL_x = {}_nN_x^* \times e^{S_x} \quad (3)$$

where,

$${}_nN_x^* = [{}_nN_x(t1) \cdot {}_nN_x(t2)]^{1/2} = \text{geometric mean of \# of people between } t1 \text{ and } t2 \quad (4)$$

$$S_x = 5 \cdot \sum_{a=0,5}^{x-5} {}_5r_a + 2.5 {}_5r_x = \text{cumulation of age-specific growth rates to midpoint of interval} \quad (5)$$

$${}_nr_x[t1, t2] = \frac{\ln({}_nN_x(t2) / {}_nN_x(t1))}{(t2 - t1)} = \text{age-specific growth rate} \quad (6)$$

The remaining functions of the life-table are the number of individuals surviving to age x (l_x), the number of person-years lived in the poor state above age x (T_x), the life expectancy of individuals exposed to mortality and net mobility in age x (e^0_x), and the combined probability of surviving and moving to or out of the poor class between age x and x+n (${}_np_x$). These functions are calculated as:

$$l_x = \frac{1}{2 \cdot n} \cdot ({}_n L_x + {}_n L_{x-n}) \quad (7)$$

l_0 = mean number of births entering the poor population between t_1 and t_2 as a result of social mobility. This radix is calculated iteratively to approximate projected and observed censal populations in the first two age groups. (8)

$$T_x = \sum_{a=x}^{\infty} {}_n L_a \quad (9)$$

$$e_x^o = \frac{T_x}{l_x} \quad (10)$$

$${}_n P_x = \frac{{}_n L_x}{{}_n L_{x-n}} \quad (11)$$

$${}_{\infty} P_x = \frac{{}_{\infty} L_x}{({}_n L_{x-n} + {}_{\infty} L_x)} \quad (12)$$

All these functions are calculated for the poor population in Table 1. Column (10), in particular, represents the values in the subdiagonal of Leslie matrix **R**. It accounts for the reproduction and mobility, of the poor class. Again, some values are higher than one to reflect the entrance of individuals into the poor class at those ages. The entrance of individuals contributes to increase life expectancies to values that are much higher than they would be in the absence of social mobility. When social mobility is taken into account, life expectancy at birth shifts to 90.17 years. This value is almost 50 percent higher than the life expectancy of 60.93 years estimated in the absence of mobility reported in the last chapter.

Table 1. Life table for the poor population of Brazil corresponding to mortality and net mobility rates in 1980 and 1990 using the r-method: $l_0= 1,752,557$

| R-method | | | | | | | | | | |
|----------|------------------|------------------|-------------|-----------|-------|------------|-----------|-------------|-------|-----------|
| | ${}_nN_x$ (1980) | ${}_nN_x$ (1990) | ${}_nN_x^*$ | ${}_nr_x$ | S_x | ${}_nL_x$ | l_x | T_x | e_x | ${}_nP_x$ |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| 0 | 7,486,155 | 8,663,037 | 8,053,126 | 0.015 | 0.037 | 8,352,516 | 1,752,557 | 158,035,904 | 90.17 | |
| 5 | 7,075,275 | 8,938,082 | 7,952,320 | 0.023 | 0.131 | 9,069,321 | 1,742,184 | 149,683,388 | 85.92 | 1.086 |
| 10 | 6,051,245 | 8,190,257 | 7,039,975 | 0.030 | 0.266 | 9,181,011 | 1,825,033 | 140,614,067 | 77.05 | 1.012 |
| 15 | 3,979,415 | 5,901,452 | 4,846,063 | 0.039 | 0.440 | 7,522,402 | 1,670,341 | 131,433,056 | 78.69 | 0.819 |
| 20 | 2,396,900 | 4,250,221 | 3,191,764 | 0.057 | 0.681 | 6,309,189 | 1,383,159 | 123,910,654 | 89.59 | 0.839 |
| 25 | 2,170,915 | 3,820,620 | 2,879,973 | 0.057 | 0.966 | 7,566,488 | 1,387,568 | 117,601,465 | 84.75 | 1.199 |
| 30 | 2,147,125 | 3,468,164 | 2,728,843 | 0.048 | 1.227 | 9,309,317 | 1,687,580 | 110,034,977 | 65.20 | 1.230 |
| 35 | 1,977,920 | 2,981,957 | 2,428,595 | 0.041 | 1.450 | 10,349,709 | 1,965,903 | 100,725,660 | 51.24 | 1.112 |
| 40 | 1,677,740 | 2,446,579 | 2,026,012 | 0.038 | 1.647 | 10,513,488 | 2,086,320 | 90,375,951 | 43.32 | 1.016 |
| 45 | 1,173,935 | 1,872,958 | 1,482,812 | 0.047 | 1.858 | 9,503,216 | 2,001,670 | 79,862,463 | 39.90 | 0.904 |
| 50 | 931,155 | 1,511,877 | 1,186,504 | 0.048 | 2.096 | 9,647,136 | 1,915,035 | 70,359,247 | 36.74 | 1.015 |
| 55 | 686,990 | 1,189,421 | 903,947 | 0.055 | 2.354 | 9,516,828 | 1,916,396 | 60,712,111 | 31.68 | 0.986 |
| 60 | 544,360 | 1,005,328 | 739,771 | 0.061 | 2.645 | 10,414,727 | 1,993,155 | 51,195,284 | 25.69 | 1.094 |
| 65 | 502,030 | 815,921 | 640,013 | 0.049 | 2.919 | 11,859,732 | 2,227,446 | 40,780,557 | 18.31 | 1.139 |
| 70 | 332,555 | 560,997 | 431,929 | 0.052 | 3.172 | 10,299,161 | 2,215,889 | 28,920,825 | 13.05 | 0.868 |
| 75 | 200,345 | 381,727 | 276,545 | 0.064 | 3.463 | 8,829,239 | 1,912,840 | 18,621,664 | 9.74 | 0.857 |
| 80 | 131,990 | 327,696 | 207,973 | 0.091 | 3.852 | 9,792,424 | 1,862,166 | 9,792,424 | 5.26 | 0.526 |

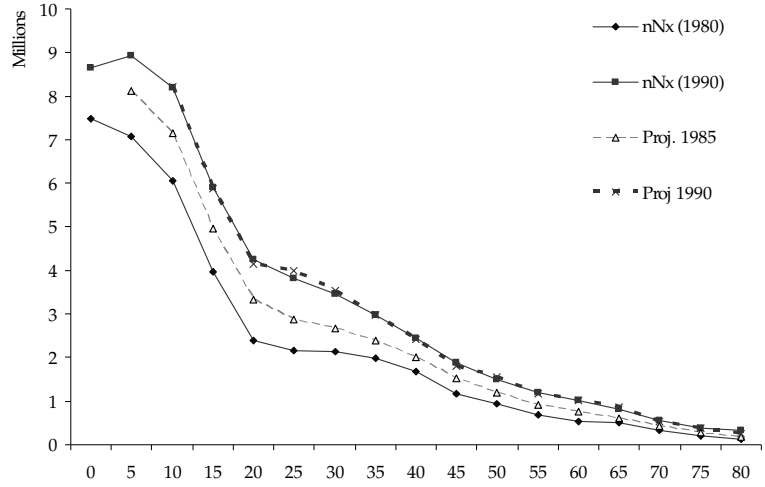
The values in column (10), when multiplied by the baseline population in 1980, provide the projected poor population in 1985. When this new projected population for 1985 is multiplied again for the respective survivorship ratio in column (10) it provides the projected population for 1990. For instance:

$${}_nPr oj(t2)_x = {}_nN_{x-2n}(t1) \cdot {}_nP_{x-n} \cdot {}_nP_x \quad (13)$$

Equation (13) shows how the number of people at age x in 1980 increased, or decreased, ten years later as a consequence of the combined effect of mortality and net mobility into that age group. Projected populations using the survivorship probabilities from column (10) are reported and plotted in table 3.2:

Table 2. Observed and projected poor populations resulting from the survivorship ratios estimated according to the r-method

| | nN_x (1980) | nN_x (1990) | Proj. 1985 | Proj 1990 |
|----|---------------|---------------|------------|-----------|
| | (1) | (2) | (11) | (12) |
| 0 | 7,486,155 | 8,663,037 | | |
| 5 | 7,075,275 | 8,938,082 | 8,128,610 | |
| 10 | 6,051,245 | 8,190,257 | 7,162,408 | 8,228,715 |
| 15 | 3,979,415 | 5,901,452 | 4,958,049 | 5,868,473 |
| 20 | 2,396,900 | 4,250,221 | 3,337,615 | 4,158,414 |
| 25 | 2,170,915 | 3,820,620 | 2,874,556 | 4,002,737 |
| 30 | 2,147,125 | 3,468,164 | 2,670,953 | 3,536,667 |
| 35 | 1,977,920 | 2,981,957 | 2,387,084 | 2,969,454 |
| 40 | 1,677,740 | 2,446,579 | 2,009,219 | 2,424,858 |
| 45 | 1,173,935 | 1,872,958 | 1,516,521 | 1,816,148 |
| 50 | 931,155 | 1,511,877 | 1,191,714 | 1,539,488 |
| 55 | 686,990 | 1,189,421 | 918,577 | 1,175,616 |
| 60 | 544,360 | 1,005,328 | 751,807 | 1,005,244 |
| 65 | 502,030 | 815,921 | 619,888 | 856,117 |
| 70 | 332,555 | 560,997 | 435,970 | 538,320 |
| 75 | 200,345 | 381,727 | 285,092 | 373,747 |
| 80 | 131,990 | 327,696 | 174,762 | 241,820 |



The values in columns (12) and (2) are very similar to each other, but they are not identical. The similarity between projected and observed poor populations suggests that the values in column (10), in Table 1, are accurate enough to represent the joint effect of mortality and mobility on population growth. There is, however, an alternative method to generate survivorship probabilities that are even more precise and perhaps even simpler than the r-method. This alternative method consists in calculating a new set of life table functions, which in the case of five-year age groups can be defined as:

$${}_n P_5^* = \frac{{}_n N_5^*}{{}_n N_{5-n}(t1)} \quad (14)$$

$${}_n P_x^* = \frac{{}_n N_x(t2)}{{}_n N_{x-2n}(t1) \cdot {}_n P_{x-n}^*} \quad \forall x > 5 \quad (15)$$

$${}_\infty P_x^* = \frac{{}_\infty L_x^{NEW}}{({}_n L_{x-n}^{NEW} + {}_\infty L_x^{NEW})} \quad (16)$$

Where:

$${}_nL_0^{NEW} = {}_nL_0 \text{ in Table 1 defined by the r-method} \quad (17)$$

$${}_nL_x^{NEW} = {}_n p_x * {}_nL_{x-n}^{NEW} \quad (18)$$

The function ${}_xL_x^{NEW}$ is defined through mathematical iterations to make projected and observed populations in t(2) to have the same size in the last age-group. The inputs and calculations of these functions for the Brazilian poor population using 1980 and 1990 censuses are in Table 3.

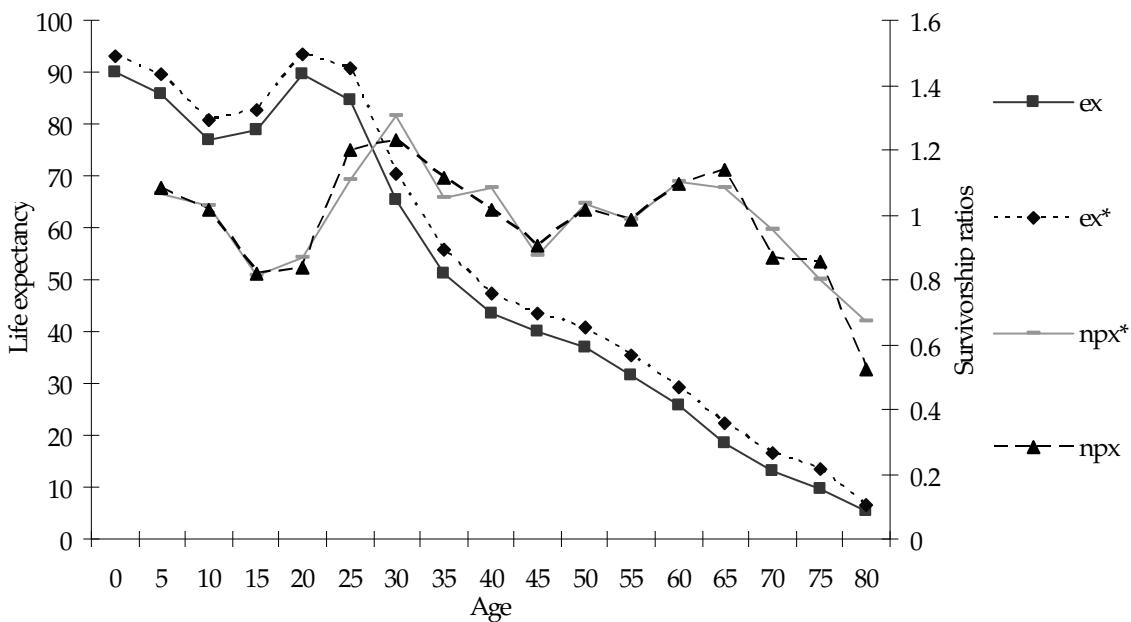
Table 3. Alternative life-table estimates providing identical observed and projected poor populations in 1990: $l_0= 1,752,557$ and $L_0= 8,352,516$

| | ${}_nN_x$ (1980) | ${}_nN_x$ (1990) | ${}_nN_x^*$ | ${}_n p_x^*$ | ${}_nL_x^{NEW}$ | l_x^* | T_x^* | e_x^* | Proj. 1985* | Proj 1990* |
|----|------------------|------------------|-------------|--------------|-----------------|-----------|-------------|---------|-------------|------------|
| | (1) | (2) | (3) | (13) | (14) | (15) | (16) | (17) | (18) | (19) |
| 0 | 7,486,155 | 8,663,037 | 8,053,126 | | 8,352,516 | 1,752,557 | 162,812,365 | 92.90 | | |
| 5 | 7,075,275 | 8,938,082 | 7,952,320 | 1.062 | 8,872,629 | 1,722,515 | 154,459,849 | 89.67 | 7,952,320 | |
| 10 | 6,051,245 | 8,190,257 | 7,039,975 | 1.030 | 9,138,102 | 1,801,073 | 145,587,219 | 80.83 | 7,286,970 | 8,190,257 |
| 15 | 3,979,415 | 5,901,452 | 4,846,063 | 0.810 | 7,400,617 | 1,653,872 | 136,449,117 | 82.50 | 4,900,683 | 5,901,452 |
| 20 | 2,396,900 | 4,250,221 | 3,191,764 | 0.867 | 6,418,342 | 1,381,896 | 129,048,501 | 93.39 | 3,451,232 | 4,250,221 |
| 25 | 2,170,915 | 3,820,620 | 2,879,973 | 1.107 | 7,105,302 | 1,352,364 | 122,630,159 | 90.68 | 2,653,442 | 3,820,620 |
| 30 | 2,147,125 | 3,468,164 | 2,728,843 | 1.307 | 9,286,939 | 1,639,224 | 115,524,857 | 70.48 | 2,837,480 | 3,468,164 |
| 35 | 1,977,920 | 2,981,957 | 2,428,595 | 1.051 | 9,759,805 | 1,904,674 | 106,237,918 | 55.78 | 2,256,451 | 2,981,957 |
| 40 | 1,677,740 | 2,446,579 | 2,026,012 | 1.084 | 10,582,162 | 2,034,197 | 96,478,113 | 47.43 | 2,144,579 | 2,446,579 |
| 45 | 1,173,935 | 1,872,958 | 1,482,812 | 0.873 | 9,241,884 | 1,982,405 | 85,895,951 | 43.33 | 1,465,247 | 1,872,958 |
| 50 | 931,155 | 1,511,877 | 1,186,504 | 1.032 | 9,536,000 | 1,877,788 | 76,654,067 | 40.82 | 1,211,295 | 1,511,877 |
| 55 | 686,990 | 1,189,421 | 903,947 | 0.982 | 9,363,801 | 1,889,980 | 67,118,066 | 35.51 | 914,340 | 1,189,421 |
| 60 | 544,360 | 1,005,328 | 739,771 | 1.100 | 10,295,611 | 1,965,941 | 57,754,265 | 29.38 | 755,354 | 1,005,328 |
| 65 | 502,030 | 815,921 | 640,013 | 1.080 | 11,121,158 | 2,141,677 | 47,458,654 | 22.16 | 588,009 | 815,921 |
| 70 | 332,555 | 560,997 | 431,929 | 0.954 | 10,610,271 | 2,173,143 | 36,337,496 | 16.72 | 478,968 | 560,997 |
| 75 | 200,345 | 381,727 | 276,545 | 0.797 | 8,456,158 | 1,906,643 | 25,727,225 | 13.49 | 265,039 | 381,727 |
| 80 | 131,990 | 327,696 | 207,973 | 0.671 | 17,271,067 | 2,572,722 | 17,271,067 | 6.71 | 223,101 | 327,696 |

Table 3 shows that projected and recorded poor populations in 1990 are now identical. The survivorship ratios reported in column (13), when multiplied by the baseline population in column (1), produce a population that is equal in size to what was indeed observed in the census. The life expectancies using this alternative methodology are also slightly higher than what was

reported in Table 1, using the r-method. For the sake of comparison, the graph below plots the life expectancies (columns 9 and 17) and the combined probability of surviving and moving in or out of the poor class between age x and $x+n$ (columns 10 and 13) according to the two methodologies:

Graph 1. Life expectancies and survivorship ratios of the poor population between 1980 and 1990 according to the r-method and according to a more accurate methodology



Graph 1 confirms that both methodologies provide very similar life expectancies and survivorship ratios, but since ${}_n p_x^*$ is more precise than ${}_n p_x$ in terms of projection outcomes, in the following analyses I use ${}_n p_x^*$ instead of the later. Life tables and projected populations for the middle and rich classes between 1980 and 2000 are in Appendix B.

RESULTS.

I introduce the main results in two sections. I first compare the fertility and survivorship ratios of the three classes with and without considering net mobility. This comparison gives a sense of how net mobility impacts the growth of income classes and how this impact has changed since 1980. In the second set of results I show how net mobility changes the net reproduction and intrinsic growth rates of income specific populations in each year. This analysis parallels the results presented in the last chapter but it incorporates the role of net mobility and how it affects population growth and distribution.

Fertility comparisons with and without mobility.

As described in the previous chapter, fertility rates were calculated using the average fertility resulting from the own children and Brass's P/F indirect methods. In this chapter, the assumption is that level and standard of fertility are the same for both cases, with and without mobility. The difference between these two scenarios lies in assumption regarding the number of births from the stationary population (l_0) in the preceding 5-year period that are entering in the projected population. According to Preston et al (2001: 122), the number of persons aged 0 to 4 at the end of the projection interval can be obtained by surviving the births through time $t+5$ following equation:

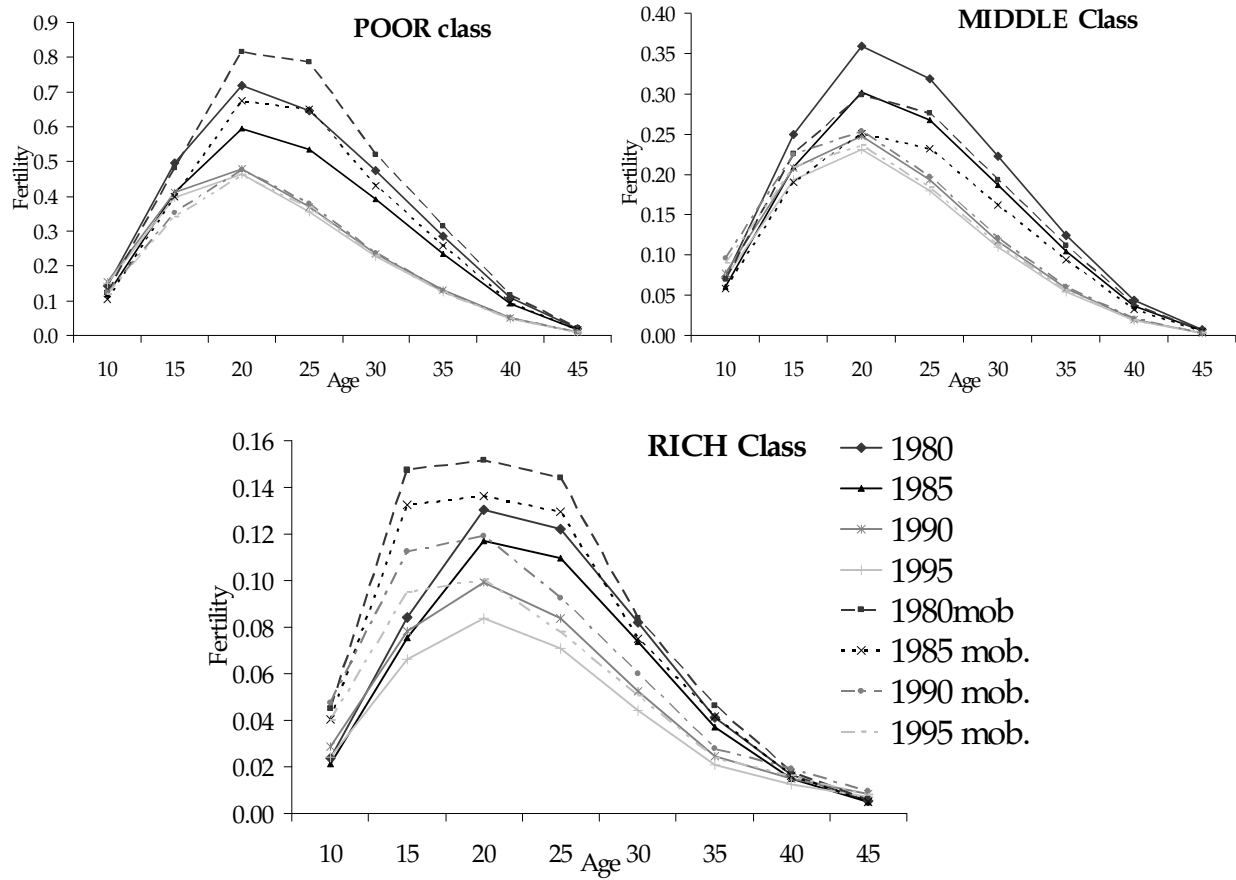
$${}_5N_0(t+5) = \frac{B[t, t+5] \cdot {}_5L_0}{5 \cdot l_0} \quad (19)$$

where $B[t, t+5]$ represents the total number of births in the population between times t and $t+5$.

Equation (19) shows that one way to “adjust” the number of individuals projected in the first age groups is to vary the number of births surviving into the future via l_0 . When net mobility is not considered in survivorship ratios, the radix of the stationary population so far has always been equal to $l_0 = 100,000$. In the scenarios with net mobility, however, the value of l_0 varies by year and income class to accommodate the number of births required to provide identical projected and observed populations⁷. Changing the value of the radix works as a “correction factor” for the number of births after including the migrants into the projection model. This correction can be understood as the impact that migrants have on the fertility of the population. The graphs below compares the entries in first row of matrices **S** and **R**, which respectively reflect fertility before and after the inclusion of net mobility in the Leslie matrices:

⁷ The values of l_0 in the scenario with net mobility are calculated iteratively and are reported in Table 3 and in Appendix B.

Figure 4. Net fertility by income class and year, with and without net mobility



The traced lines in Figure 4 represent fertility levels between 1980 and 1995 incorporating the impact of net mobility and after discounting the influence of mortality (e.g. the first row of the Leslie matrices of each class). The three graphs in the figure have different scales to emphasize and facilitate the visualization of the differences in fertility with and without considering net mobility. The overall result is that the rise and fall of fertility follows the pattern of net mobility in the 1980s and 1990s. Net fertility is higher than previously estimated values when net mobility is positive and lower when it is negative.

In the 1980s, a period of intense downward mobility to the poor class, fertility rates after considering mobility were higher than in the previous chapter, where fertility is estimated

without considering the impact of net mobility. At the end of the 1980s, after considering mobility the projected number of births contributing to population growth in the poor class was about nine percent higher than in the scenario without mobility. In the 1990s, this dynamic was the opposite since there were more individuals leaving than entering into the poor class. As a result, the net fertility of the poor class was slightly lower than in the case without mobility. The projected number of births in the 1990s, after considering the exits of the poor class, was 81 percent of the total number of births in the scenario without mobility.

In the rich class the impact of net mobility on fertility was even more eminent. The graph of the rich class in Figure 4 shows that the fertility of the rich population is clearly higher once mobility is considered. Because the rich class received migrants from other classes during all years, its net fertility is clearly higher after incorporating migrants' births. In 1990, there were 52 percent more births in the projected rich population with mobility than in the projection without. In 2000 this figure was similar, shifting to 48 percent.

Mortality comparisons with and without mobility.

Mortality is examined by comparing two indicators: survivorship ratios and life expectancies. Comparing survivorships with and without net mobility will indicate at which age groups mobility is more, or less, prevalent for each income class. The other indicator, life expectancy, will show how much longer one would, on average, expect to live if patterns of mortality and net mobility were to remain constant over the life cycle. I borrow the class specific mortality estimates from the previous chapter and compare them to what would be observed had net mobility been considered.

Figure 5. Survivorship ratios by income class, Brazil, 1980s and 1990s

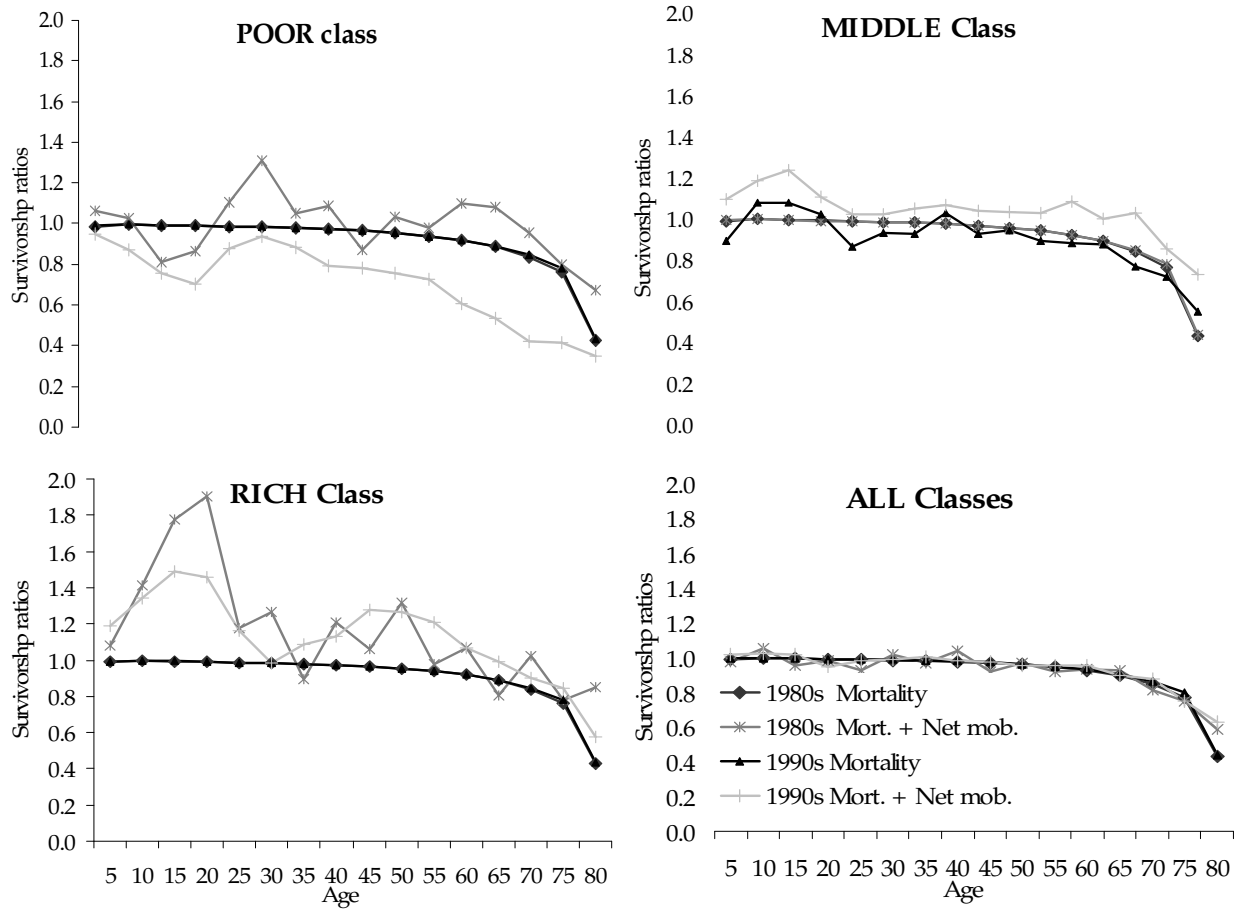


Figure 5 plots the survivorship ratios of each income class. The entries in the graphs correspond to the subdiagonal of matrices \mathbf{R} (mortality plus net mobility) and \mathbf{S} (only mortality). The graphs show the age pattern of net mobility in each income class. Values below one imply in exit from that class through death or mobility, and values above one correspond to ages where the flow of migrants from other classes was positive. The first thing to notice is that the curves have similar patterns by age, but different levels.

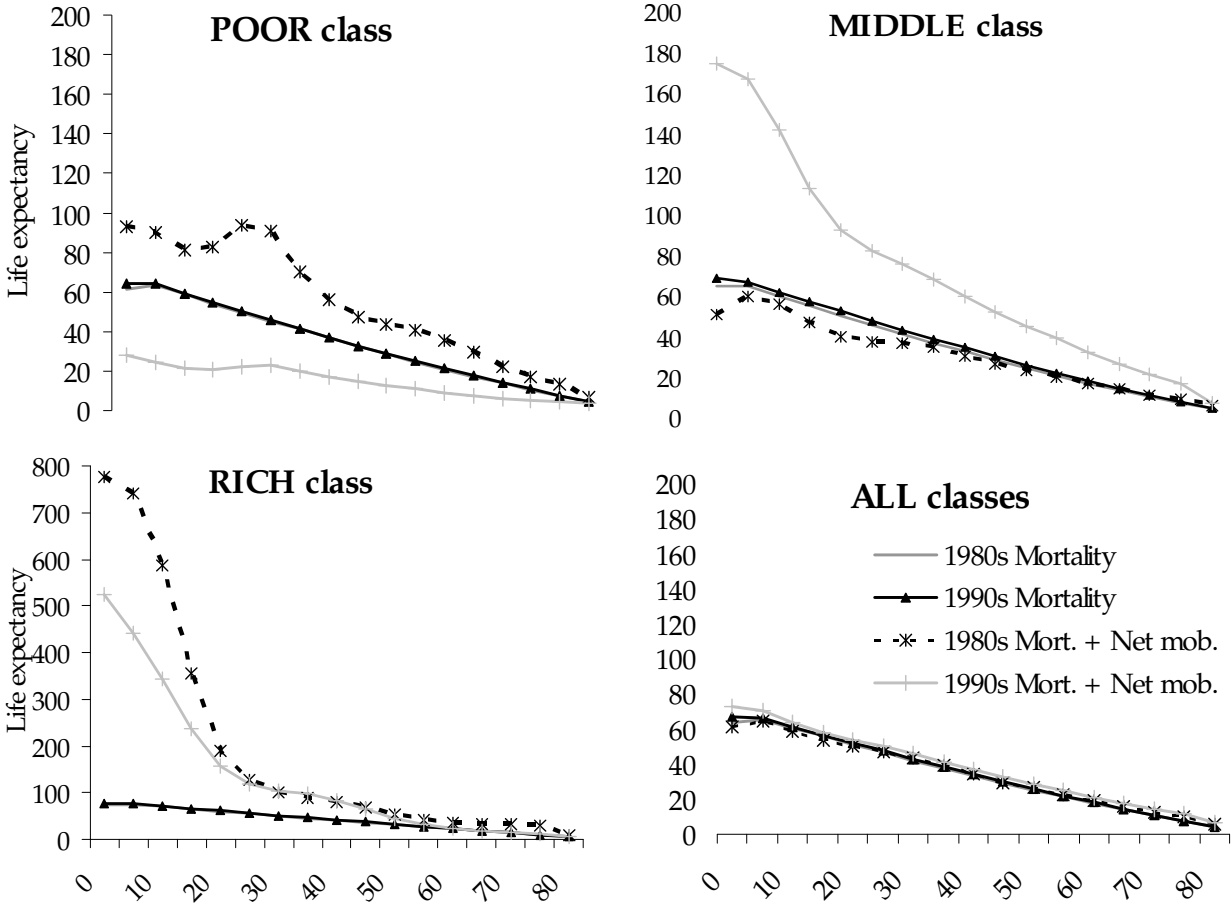
The net mobility standard of the poor class shows that the peak of entrance into that same group happened in the 1980s at ages 25 and 60. Yet, during the 1990s there was a reversal in this flow and the exit from the poor class could be perceived at all ages, but it was particularly high

after ages 50 and above. The counterpart of this exit of the poor class is reflected by the entrance into the middle class, where survivorship was higher than one at all ages during the 1990s. The rich class had entrants during both decades, but the proportion of people moving into the rich class was particularly marked during the first phase of the life cycle, between ages 5 and 20. It is also worth noticing that mobility has its most obvious impact on the growth of the rich class, where the discrepancy between the survivorship curves is most evident. Finally, the small differences in the curves with and without net mobility, in the lower right graph (e.g. ALL Classes), are attributable to international migration.

Another way of measuring the effect of net mobility on survival is to compare “life expectancies”. Life expectancies after mobility indicate the average duration in a certain income class if mobility and mortality patterns were to remain indefinitely constant in a synthetic cohort. The cautionary tale is that positive net mobility generates biologically impossible “life expectancies”. Values higher than 100 years are obviously hypothetical and unrealistic, but they are useful to signalize the relative impact of mobility in comparison to scenarios where mortality is the only source of decrement. Figure 6 plots life expectancies of income classes in the 1980s and 1990s with and without considering the impact of net mobility.

Figure 6 shows that positive net mobility in the 1980s contributed to increase the average duration of years spent in the poor and in the rich classes, especially before age 30. The middle and rich classes received all the migrants who left the middle class during the 1980s. In the 1990s, however, because of upward mobility from the poor to the middle class, life expectancies increased significantly in the middle class, especially before age 30.

Figure 6. Life expectancies by income class, Brazil, 1980s and 1990s



Net reproduction and intrinsic growth rates under the influence of mobility.

This section describes how poor, middle and rich classes have grown and reproduced after considering the joint effect of fertility, mortality and net mobility. Table 4 shows how the population dynamics of each income class changes after including net mobility in the calculations of period measures of reproduction and stable growth.

In terms of reproduction, Table 4 indicates that net mobility had two effects over time. In the 1980s, as noticed before, it contributed to reinforce the growth of the poor and rich classes. Net mobility increased the net reproduction rate (NRR) of the poor by about 8 percent and the

NRR of the rich by 560 percent in relation to the scenario where only fertility and mortality are considered. Net mobility to the rich class reversed the NRR from .49 to 3.28, which indicates that the reason why the rich class is not following a path of extinction is because of upward mobility from other classes. For the middle class, net mobility helped to decrease the NRR from 1.34 to 1.15 and to bring this class closer to replacement.

In the 1990s, net mobility reversed all the period indicators discussed in the last chapter. On the one hand, the massive exit from the poor class reduced the net reproduction of this class by 72 percent between 1980 and 1990, from 2.97 to .82. On the other hand, the entrance of individuals into the middle and rich classes increased their growth and reversed their NRR to values above replacement. The NRR of the middle class shifted from .90 to 1.67, and the NRR of the rich class shifted from .39 to 1.64 daughters for every rich woman after considering net mobility. This shift is also reflected in the crude rate of natural increase (CRNI), which increased from .0015 to .0349 in the rich class.

Table 4. Comparison between observed and stable-equivalent population parameters under two scenarios: with and without considering the influence of net mobility

| WITHOUT CONSIDERING NET MOBILITY | | | | | | | |
|----------------------------------|------------|------------------------|----------|----------|--------------------|------------|------------|
| <i>Year, Social Class</i> | <i>NRR</i> | <i>Intrinsic rates</i> | | | <i>Crude rates</i> | | |
| | | <i>r</i> | <i>b</i> | <i>d</i> | <i>CRNI</i> | <i>CBR</i> | <i>CDR</i> |
| 1980 | | | | | | | |
| Poor | 2.7583 | 0.0366 | 0.0462 | 0.0096 | 0.0283 | 0.0385 | 0.0102 |
| Middle | 1.3476 | 0.0107 | 0.0223 | 0.0116 | 0.0179 | 0.0266 | 0.0087 |
| Rich | 0.4973 | -0.0239 | 0.0042 | 0.0281 | 0.0050 | 0.0102 | 0.0053 |
| 1990 | | | | | | | |
| Poor | 1.7729 | 0.0217 | 0.0310 | 0.0093 | 0.0202 | 0.0288 | 0.0086 |
| Middle | 0.9035 | -0.0038 | 0.0126 | 0.0163 | 0.0101 | 0.0180 | 0.0079 |
| Rich | 0.3863 | -0.0329 | 0.0024 | 0.0353 | 0.0015 | 0.0078 | 0.0063 |

| CONSIDERING NET MOBILITY | | | | | | | |
|-------------------------------|------------|------------------------|----------|----------|--------------------|------------|------------|
| <i>Year, Social Class</i> | <i>NRR</i> | <i>Intrinsic rates</i> | | | <i>Crude rates</i> | | |
| | | <i>r</i> | <i>b</i> | <i>d</i> | <i>CRNI</i> | <i>CBR</i> | <i>CDR</i> |
| 1980 | | | | | | | |
| Poor | 2.9730 | 0.0377 | 0.0399 | 0.0022 | 0.0370 | 0.0385 | 0.0015 |
| Middle | 1.1511 | 0.0051 | 0.0233 | 0.0182 | 0.0125 | 0.0266 | 0.0141 |
| Rich | 3.2843 | 0.0406 | 0.0098 | -0.031 | 0.0501 | 0.0102 | -0.040 |
| 1990 | | | | | | | |
| Poor | 0.8167 | -0.0079 | 0.0303 | 0.0382 | -0.012 | 0.0288 | 0.0413 |
| Middle | 1.6700 | 0.0192 | 0.0137 | -0.005 | 0.0325 | 0.0180 | -0.014 |
| Rich | 1.6386 | 0.0171 | 0.0046 | -0.012 | 0.0349 | 0.0078 | -0.027 |

The impact of net mobility on future growth can be inferred by examining intrinsic rates. The trends in Table 4 show that if fertility, mortality and net mobility remained constant for a very long time, the poor class would decrease, while the middle and rich classes would increase. Between 1980 and 1990 the rhythm of growth of the middle class increased; the rhythm of the rich decreased, but remained positive; and the rhythm of growth of the poor became negative in the 1990s.

The influence of mobility on the reproduction of poverty was pervasive in the 1980s and 1990s, but it had opposite effects during these two periods. In the first period it helped to

increase poverty, but in the second it acted on the opposite direction, contributing to alleviate poverty and its reproduction. Net mobility was particularly important in the reproduction of the rich class, but since this class accounts for only about one percent of the population its overall influence is not so important to one concerned with absolute numbers.

SUMMARY AND CONCLUSION.

This chapter showed how to measure the influence of net mobility on fertility, mortality and growth of specific income classes using Brazilian census data. Two questions guided this chapter. The first asked how one can measure intragenerational flows of net mobility in the absence of longitudinal or retrospective information. The solution I offered is based on a variation of the r-method using intercensal survivorship. Survivorship ratios estimated according to this variation incorporate the joint effect of mortality and net mobility, but they also present a “correction” for the number of expected births in the presence of mobility.

One important methodological contribution of this chapter is to show how life tables estimated with different methodologies (with and without net mobility) provide different projected population sizes and age structures. In particular, it advances our knowledge of projection methods by describing how the future growth and distribution of subpopulations may differ under scenarios with and without net mobility. The behavior of net mobility is hard to predict because it depends on market forces and other trends that are not intuitive and easy to estimate, but the results of this chapter signalize that the curves of net mobility have a similar age pattern in each class, despite their differences in levels over time. If these patterns turn out to be an empirical regularity rather than a data coincidence, they could be estimated and incorporated into future population projections by income level. More data and further empirical validations

considering other time periods would, however, be required before making final assertions on this matter.

The second goal of this chapter was to demonstrate how net mobility affects the reproduction of poverty in comparison to a scenario where only fertility and mortality are considered. The results show that the impact of net mobility depends on the period of analysis. In the 1980s, Brazil was plagued by economic crisis and very high inflation. During this period the average time spent in poverty would be 59.5 years if mortality was the only factor contributing to this figure. After considering the entrance of movers to the poor class, however, the number of years spent in poverty would increase to 92.9 if mortality and mobility conditions were to remain the same over the life cycle. In the 1990s, a period marked by considerable economic improvements and social upward mobility from the poor class, the “life expectancy” in poverty shifted from 62.6 to 27.8 years, before and after considering net mobility.

The mobility dynamics of the poor, middle and rich classes between 1980 and 2000 also altered the speed of reproduction of these groups. The average number of daughters being “born” in the poor class in 1980 shifted from 2.76 to 2.97 due to positive mobility. In 1990, however, the net reproduction of the poor declined from 1.77 to .82 after considering the exit of individuals of that class.

The period indicators reported in this chapter (NRR , e_0) are useful to predict and understand what would happen to poverty and to the size and distribution of other economic classes if demographic circumstances were to remain stable over time. When mortality, fertility and mobility conditions change, however, little can be said about what will happen. One way to address this problem is to produce “what if” scenarios.

The results presented in this and in the previous chapters set an empirical base to experiment and conduct counterfactual analysis dealing with different scenarios. In the next chapter I investigate the possible paths that poverty and inequality would follow had the demographic terms been different.

Appendix A. Leslie matrices with and without net mobility, baseline and projected populations
by income class and year

- Projection models for the Brazilian **poor, 1980-1985**

Matrix including fertility and mortality

| $w^{t+5}_{\text{POOR}^*}$ Poor^{1985} | S_{POOR} | | | | | | | | | | | | | | w^t_{POOR} Poor^{1980} | | | |
|---|-------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|---|------|-----------|---------|
| 7,760,538 | 0 | 0 | 0.14 | 0.50 | 0.72 | 0.65 | 0.47 | 0.28 | 0.11 | 0.02 | 0 | 0 | 0 | 0 | 0 | 0 | 7,486,155 | |
| 7,396,204 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,075,275 | |
| 7,043,996 | 0 | 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6,051,245 | |
| 6,012,191 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,979,415 | |
| 3,936,405 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,396,900 | |
| 2,362,833 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,170,915 | |
| 2,133,777 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,147,125 | |
| 2,101,456 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1,977,920 | |
| 1,924,381 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.97 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1,677,740 | |
| 1,618,401 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.96 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1,173,935 | |
| 1,119,114 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.95 | 0 | 0 | 0 | 0 | 0 | 0 | 931,155 | |
| 873,685 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.94 | 0 | 0 | 0 | 0 | 0 | 686,990 | |
| 630,254 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.92 | 0 | 0 | 0 | 0 | 544,360 | |
| 483,437 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.89 | 0 | 0 | 0 | 502,030 | |
| 420,402 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.84 | 0 | 0 | 332,555 | |
| 252,756 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.76 | 0 | 200,345 | |
| 141,623 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.43 | 0.43 | 131,990 |

Matrix including fertility and (mortality+ net mobility)

| w^{t+5}_{POOR} Poor^{1985} | $R_t = S_t + M_t$ | | | | | | | | | | | | | | w^t_1 Poor^{1980} | | | |
|---|-------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|---------------------------------|------|-----------|---------|
| 8,288,408 | 0 | 0 | 0.12 | 0.48 | 0.81 | 0.79 | 0.52 | 0.31 | 0.12 | 0.02 | 0 | 0 | 0 | 0 | 0 | 0 | 7,486,155 | |
| 7,952,320 | 1.06 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,075,275 | |
| 7,286,970 | 0 | 1.03 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6,051,245 | |
| 4,900,683 | 0 | 0 | 0.81 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,979,415 | |
| 3,451,232 | 0 | 0 | 0 | 0.87 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,396,900 | |
| 2,653,442 | 0 | 0 | 0 | 0 | 1.11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,170,915 | |
| 2,837,480 | 0 | 0 | 0 | 0 | 0 | 1.31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,147,125 | |
| 2,256,451 | 0 | 0 | 0 | 0 | 0 | 0 | 1.05 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1,977,920 | |
| 2,144,579 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.08 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1,677,740 | |
| 1,465,247 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.87 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1,173,935 | |
| 1,211,295 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.03 | 0 | 0 | 0 | 0 | 0 | 0 | 931,155 | |
| 914,340 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 686,990 | |
| 755,354 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.10 | 0 | 0 | 0 | 0 | 544,360 | |
| 588,009 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.08 | 0 | 0 | 0 | 502,030 | |
| 478,968 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.95 | 0 | 0 | 332,555 | |
| 265,039 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.80 | 0 | 200,345 | |
| 223,101 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.67 | 0.67 | 131,990 |

Appendix A. Cont'd.

- Projection models for the Brazilian **poor, 1990-1995**

Matrix including fertility and mortality

| w^{t+5}_{POOR} Poor ¹⁹⁹⁵ | S_{POOR} | | | | | | | | | | | | | | w^t_{POOR} Poor ¹⁹⁹⁰ | | |
|--|------------|------|------|------|------|------|------|------|------|------|------|------|------|------|--------------------------------------|-----------|-----------|
| 8,490,708 | 0 | 0 | 0.16 | 0.41 | 0.48 | 0.37 | 0.23 | 0.13 | 0.05 | 0.01 | 0 | 0 | 0 | 0 | 0 | 0 | 8,663,037 |
| 8,606,705 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,938,082 |
| 8,907,027 | 0 | 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,190,257 |
| 8,138,694 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5,901,452 |
| 5,834,772 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4,250,221 |
| 4,187,988 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,820,620 |
| 3,755,289 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,468,164 |
| 3,397,077 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,981,957 |
| 2,904,756 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.97 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,446,579 |
| 2,362,948 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.97 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1,872,958 |
| 1,787,476 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.95 | 0 | 0 | 0 | 0 | 0 | 0 | 1,511,877 |
| 1,419,355 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.94 | 0 | 0 | 0 | 0 | 0 | 1,189,421 |
| 1,091,616 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.92 | 0 | 0 | 0 | 0 | 1,005,328 |
| 893,265 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.89 | 0 | 0 | 0 | 815,921 |
| 688,562 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.84 | 0 | 0 | 560,997 |
| 436,350 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.78 | 0 | 381,727 |
| 307,311 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.43 0.43 | 327,696 |

Matrix including fertility and (mortality+ net mobility)

| w^{t+5}_{POOR} Poor ¹⁹⁹⁵ | $R_t = S_t + M_t$ | | | | | | | | | | | | | | w^t_{POOR} Poor ¹⁹⁹⁰ | | |
|--|-------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|--------------------------------------|-----------|-----------|
| 7,919,805 | 0 | 0 | 0.12 | 0.35 | 0.48 | 0.38 | 0.24 | 0.13 | 0.05 | 0.01 | 0 | 0 | 0 | 0 | 0 | 0 | 8,663,037 |
| 8,219,369 | 0.95 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,938,082 |
| 7,782,508 | 0 | 0.87 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,190,257 |
| 6,196,793 | 0 | 0 | 0.76 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5,901,452 |
| 4,131,674 | 0 | 0 | 0 | 0.70 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4,250,221 |
| 3,716,587 | 0 | 0 | 0 | 0 | 0.87 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,820,620 |
| 3,574,909 | 0 | 0 | 0 | 0 | 0 | 0.94 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,468,164 |
| 3,055,618 | 0 | 0 | 0 | 0 | 0 | 0 | 0.88 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,981,957 |
| 2,367,690 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.79 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,446,579 |
| 1,916,070 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.78 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1,872,958 |
| 1,415,786 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.76 | 0 | 0 | 0 | 0 | 0 | 0 | 1,511,877 |
| 1,101,334 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.73 | 0 | 0 | 0 | 0 | 0 | 1,189,421 |
| 719,852 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.61 | 0 | 0 | 0 | 0 | 1,005,328 |
| 536,197 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.53 | 0 | 0 | 0 | 815,921 |
| 341,980 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.42 | 0 | 0 | 560,997 |
| 233,640 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.42 | 0 | 381,727 |
| 245,087 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.35 0.35 | 327,696 |

Appendix A. Cont'd.

- Projection models for the Brazilian **poor, 1995-2000**

Matrix including fertility and mortality

| age | $w^{t+10}_{\text{POOR}^*}$ Poor ²⁰⁰⁰ | S_{POOR} | | | | | | | | | | | | | | w^{t+5}_{POOR} Poor ¹⁹⁹⁵ | | | |
|-----|--|-------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|---|------|------|-----------|
| 0 | 10,212,182 | 0 | 0 | 0.15 | 0.40 | 0.46 | 0.36 | 0.23 | 0.13 | 0.05 | 0.01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,490,708 |
| 5 | 8,435,497 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,606,705 |
| 10 | 8,576,801 | 0 | 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,907,027 |
| 15 | 8,850,951 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,138,694 |
| 20 | 8,046,735 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5,834,772 |
| 25 | 5,749,337 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4,187,988 |
| 30 | 4,116,375 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,755,289 |
| 35 | 3,678,317 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,397,077 |
| 40 | 3,309,129 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.97 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,904,756 |
| 45 | 2,805,463 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.97 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,362,948 |
| 50 | 2,255,104 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.95 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1,787,476 |
| 55 | 1,678,089 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.94 | 0 | 0 | 0 | 0 | 0 | 0 | 1,419,355 |
| 60 | 1,302,643 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.92 | 0 | 0 | 0 | 0 | 0 | 1,091,616 |
| 65 | 969,934 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.89 | 0 | 0 | 0 | 0 | 893,265 |
| 70 | 753,833 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.84 | 0 | 0 | 0 | 688,562 |
| 75 | 535,571 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.78 | 0 | 0 | 436,350 |
| 80+ | 322,143 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.43 | 0.43 | 307,311 |

Matrix including fertility and (mortality+ net mobility)

| age | w^{t+10}_{POOR} Poor ²⁰⁰⁰ | $R_{\text{POOR}} = S_{\text{POOR}} + M_{\text{POOR}}$ | | | | | | | | | | | | | | w^{t+5}_{POOR} Poor ¹⁹⁹⁵ | | | |
|-----|--|---|------|------|------|------|------|------|------|------|------|------|------|------|------|---|------|------|-----------|
| 0 | 7,642,956 | 0 | 0 | 0.12 | 0.34 | 0.46 | 0.37 | 0.23 | 0.12 | 0.05 | 0.01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,919,805 |
| 5 | 7,514,200 | 0.95 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,219,369 |
| 10 | 7,156,714 | 0 | 0.87 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,782,508 |
| 15 | 5,888,288 | 0 | 0 | 0.76 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6,196,793 |
| 20 | 4,338,445 | 0 | 0 | 0 | 0.70 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4,131,674 |
| 25 | 3,612,924 | 0 | 0 | 0 | 0 | 0.87 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,716,587 |
| 30 | 3,477,567 | 0 | 0 | 0 | 0 | 0 | 0.94 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,574,909 |
| 35 | 3,149,666 | 0 | 0 | 0 | 0 | 0 | 0 | 0.88 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,055,618 |
| 40 | 2,426,178 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.79 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,367,690 |
| 45 | 1,854,287 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.78 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1,916,070 |
| 50 | 1,448,374 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.76 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1,415,786 |
| 55 | 1,031,336 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.73 | 0 | 0 | 0 | 0 | 0 | 0 | 1,101,334 |
| 60 | 666,541 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.61 | 0 | 0 | 0 | 0 | 0 | 719,852 |
| 65 | 383,937 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.53 | 0 | 0 | 0 | 0 | 536,197 |
| 70 | 224,738 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.42 | 0 | 0 | 0 | 341,980 |
| 75 | 142,425 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.42 | 0 | 0 | 233,640 |
| 80+ | 165,387 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.35 | 0.35 | 245,087 |

Appendix A. Cont'd.

- Projection models for the Brazilian **middle, 1980-1985**

Matrix including fertility and mortality

| $W^{t+5}_{MIDDLE*}$ Middle ¹⁹⁸⁵ | S_{MIDDLE} | | | | | | | | | | | | | | | W^t_{MIDDLE} Middle ¹⁹⁸⁰ | | | | | |
|---|--------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|--|------|---|---|-----------|-----------|
| 10,221,261 | 0 | 0 | 0.07 | 0.25 | 0.36 | 0.32 | 0.22 | 0.12 | 0.04 | 0.01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,813,965 |
| 8,737,159 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,595,495 |
| 7,570,137 | 0 | 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,091,295 |
| 8,051,852 | 0 | 0 | 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 9,403,650 |
| 9,326,008 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,797,045 |
| 8,699,542 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6,988,560 |
| 6,892,916 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5,314,115 |
| 5,221,001 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4,240,290 |
| 4,142,783 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,938,250 |
| 3,815,833 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.97 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,392,195 |
| 3,247,685 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.96 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,053,805 |
| 2,874,458 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.94 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,346,940 |
| 2,153,868 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.92 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1,808,335 |
| 1,597,177 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.88 | 0 | 0 | 0 | 0 | 0 | 0 | 1,464,795 | |
| 1,205,027 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.82 | 0 | 0 | 0 | 0 | 0 | 947,850 | |
| 691,371 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.73 | 0 | 0 | 0 | 0 | 601,880 | |
| 429,896 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.42 | 0.42 | 0 | 0 | 429,925 | |

Matrix including fertility and (mortality+ net mobility)

| W^{t+5}_{MIDDLE} Middle ¹⁹⁸⁵ | $R_{MIDDLE} = S_{MIDDLE} + M_{MIDDLE}$ | | | | | | | | | | | | | | | W^t_{MIDDLE} Middle ¹⁹⁸⁰ | | | | | |
|--|--|------|------|------|------|------|------|------|------|------|------|------|------|------|------|--|------|---|---|-----------|-----------|
| 8,921,968 | 0 | 0 | 0.07 | 0.23 | 0.30 | 0.28 | 0.19 | 0.11 | 0.04 | 0.01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,813,965 |
| 7,868,118 | 0.89 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,595,495 |
| 8,187,515 | 0 | 1.08 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,091,295 |
| 8,704,785 | 0 | 0 | 1.08 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 9,403,650 |
| 9,576,799 | 0 | 0 | 0 | 1.02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,797,045 |
| 7,564,913 | 0 | 0 | 0 | 0 | 0.86 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6,988,560 |
| 6,487,274 | 0 | 0 | 0 | 0 | 0 | 0.93 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5,314,115 |
| 4,900,189 | 0 | 0 | 0 | 0 | 0 | 0 | 0.92 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4,240,290 |
| 4,344,099 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,938,250 |
| 3,637,747 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.92 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,392,195 |
| 3,187,444 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.94 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,053,805 |
| 2,713,292 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.89 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2,346,940 |
| 2,069,175 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.88 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1,808,335 |
| 1,580,193 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.87 | 0 | 0 | 0 | 0 | 0 | 0 | 1,464,795 | |
| 1,125,341 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.77 | 0 | 0 | 0 | 0 | 0 | 947,850 | |
| 677,468 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.71 | 0 | 0 | 0 | 0 | 601,880 | |
| 563,874 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.55 | 0.55 | 0 | 0 | 429,925 | |

Appendix A. Cont'd.

- Projection models for the Brazilian **middle, 1985-1990**

Matrix including fertility and mortality

| age | w^{t+10} MIDDLE* Middle ¹⁹⁹⁰ | S_{MIDDLE} | | | | | | | | | | | | | | w^{t+5} MIDDLE Middle ¹⁹⁸⁵ | | | |
|-----|---|--------------|------|------|------|------|------|------|------|------|------|------|------|------|------|---|------|------|------------|
| 0 | 9,288,085 | 0 | 0 | 0.06 | 0.21 | 0.30 | 0.27 | 0.19 | 0.10 | 0.04 | 0.01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 10,221,261 |
| 5 | 10,132,192 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,737,159 |
| 10 | 8,707,989 | 0 | 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,570,137 |
| 15 | 7,533,234 | 0 | 0 | 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,051,852 |
| 20 | 7,985,371 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 9,326,008 |
| 25 | 9,222,642 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,699,542 |
| 30 | 8,580,482 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6,892,916 |
| 35 | 6,772,137 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5,221,001 |
| 40 | 5,100,942 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4,142,783 |
| 45 | 4,014,008 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.97 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,815,833 |
| 50 | 3,653,275 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.96 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,247,685 |
| 55 | 3,056,952 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.94 | 0 | 0 | 0 | 0 | 0 | 0 | 2,874,458 |
| 60 | 2,637,989 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.92 | 0 | 0 | 0 | 0 | 0 | 2,153,868 |
| 65 | 1,902,362 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.88 | 0 | 0 | 0 | 0 | 1,597,177 |
| 70 | 1,313,932 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.82 | 0 | 0 | 0 | 1,205,027 |
| 75 | 878,958 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.73 | 0 | 0 | 691,371 |
| 80+ | 467,171 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.42 | 0.42 | 429,896 |

Matrix including fertility and (mortality+ net mobility)

| age | w^{t+10} MIDDLE Middle ¹⁹⁹⁰ | $R_{MIDDLE} = S_{MIDDLE} + M_{MIDDLE}$ | | | | | | | | | | | | | | w^{t+5} MIDDLE Middle ¹⁹⁸⁵ | | | |
|-----|--|--|------|------|------|------|------|------|------|------|------|------|------|------|------|---|------|------|-----------|
| 0 | 7,957,685 | 0 | 0 | 0.06 | 0.19 | 0.25 | 0.23 | 0.16 | 0.09 | 0.03 | 0.01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,921,968 |
| 5 | 7,964,530 | 0.89 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,868,118 |
| 10 | 8,481,387 | 0 | 1.08 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,187,515 |
| 15 | 8,808,301 | 0 | 0 | 1.08 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,704,785 |
| 20 | 8,865,066 | 0 | 0 | 0 | 1.02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 9,576,799 |
| 25 | 8,235,452 | 0 | 0 | 0 | 0 | 0.86 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,564,913 |
| 30 | 7,022,285 | 0 | 0 | 0 | 0 | 0 | 0.93 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6,487,274 |
| 35 | 5,981,968 | 0 | 0 | 0 | 0 | 0 | 0 | 0.92 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4,900,189 |
| 40 | 5,020,153 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4,344,099 |
| 45 | 4,012,628 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.92 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,637,747 |
| 50 | 3,418,176 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.94 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,187,444 |
| 55 | 2,832,030 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.89 | 0 | 0 | 0 | 0 | 0 | 0 | 2,713,292 |
| 60 | 2,392,168 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.88 | 0 | 0 | 0 | 0 | 0 | 2,069,175 |
| 65 | 1,808,125 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.87 | 0 | 0 | 0 | 0 | 1,580,193 |
| 70 | 1,213,997 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.77 | 0 | 0 | 0 | 1,125,341 |
| 75 | 804,328 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.71 | 0 | 0 | 677,468 |
| 80+ | 678,384 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.55 | 0.55 | 563,874 |

Appendix A. Cont'd.

- Projection models for the Brazilian **middle, 1990-1995**

Matrix including fertility and mortality

| $w^{t+5}_{MIDDLE*}$ Middle ¹⁹⁹⁵ | S_{MIDDLE} | | | | | | | | | | | | | | w^t_{MIDDLE} Middle ¹⁹⁹⁰ | | | |
|---|--------------|-------|-------|------|------|------|------|------|------|------|------|------|------|------|--|------|------|-----------|
| 7,556,157 | 0 | 0 | 0.08 | 0.21 | 0.25 | 0.19 | 0.12 | 0.06 | 0.02 | 0.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,771,690 |
| 7,739,786 | 0.996 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,150,525 |
| 8,132,761 | 0 | 0.998 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,481,387 |
| 8,447,724 | 0 | 0 | 0.996 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,808,301 |
| 8,744,711 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,865,066 |
| 8,780,340 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,235,452 |
| 8,141,137 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,022,285 |
| 6,923,239 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5,981,968 |
| 5,872,249 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5,020,153 |
| 4,894,477 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.97 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4,012,628 |
| 3,873,065 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.97 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,418,176 |
| 3,251,658 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.95 | 0 | 0 | 0 | 0 | 0 | 0 | 2,832,030 |
| 2,637,664 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.93 | 0 | 0 | 0 | 0 | 0 | 2,392,168 |
| 2,157,487 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.90 | 0 | 0 | 0 | 0 | 1,808,125 |
| 1,545,333 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.85 | 0 | 0 | 0 | 1,213,997 |
| 949,973 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.78 | 0 | 0 | 804,328 |
| 647,676 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.44 | 0.44 | 678,384 |

Matrix including fertility and (mortality+ net mobility)

| w^{t+5}_{MIDDLE} Middle ¹⁹⁹⁵ | $R_{MIDDLE} = S_{MIDDLE} + M_{MIDDLE}$ | | | | | | | | | | | | | | w^t_{MIDDLE} Middle ¹⁹⁹⁰ | | | |
|--|--|------|------|------|------|------|------|------|------|------|------|------|------|------|--|------|------|-----------|
| 7,991,861 | 0 | 0 | 0.10 | 0.22 | 0.25 | 0.20 | 0.12 | 0.06 | 0.02 | 0.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,771,690 |
| 8,513,556 | 1.10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,150,525 |
| 9,625,562 | 0 | 1.18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,481,387 |
| 10,443,057 | 0 | 0 | 1.23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,808,301 |
| 9,724,897 | 0 | 0 | 0 | 1.10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,865,066 |
| 9,060,911 | 0 | 0 | 0 | 0 | 1.02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,235,452 |
| 8,412,995 | 0 | 0 | 0 | 0 | 0 | 1.02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,022,285 |
| 7,360,175 | 0 | 0 | 0 | 0 | 0 | 0 | 1.05 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5,981,968 |
| 6,372,693 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.07 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5,020,153 |
| 5,213,248 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.04 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4,012,628 |
| 4,133,973 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.03 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,418,176 |
| 3,498,655 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.02 | 0 | 0 | 0 | 0 | 0 | 0 | 2,832,030 |
| 3,063,055 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.08 | 0 | 0 | 0 | 0 | 0 | 2,392,168 |
| 2,383,446 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.00 | 0 | 0 | 0 | 0 | 1,808,125 |
| 1,850,075 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.02 | 0 | 0 | 0 | 1,213,997 |
| 1,030,389 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.85 | 0 | 0 | 804,328 |
| 1,079,725 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.73 | 0.73 | 678,384 |

Appendix A. Cont'd.

- Projection models for the Brazilian **middle, 1995-2000**

Matrix including fertility and mortality

| age | w^{t+10} MIDDLE* Middle ²⁰⁰⁰ | S_{MIDDLE} | | | | | | | | | | | | | | w^{t+5} MIDDLE Middle ¹⁹⁹⁵ | | | | |
|-----|---|--------------|-------|-------|------|------|------|------|------|------|------|------|------|------|------|---|------|------|---|-----------|
| 0 | 7,197,699 | 0 | 0 | 0.07 | 0.19 | 0.23 | 0.18 | 0.11 | 0.05 | 0.02 | 0.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,556,157 |
| 5 | 7,525,137 | 0.996 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,739,786 |
| 10 | 7,722,917 | 0 | 0.998 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,132,761 |
| 15 | 8,100,482 | 0 | 0 | 0.996 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,447,724 |
| 20 | 8,386,737 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,744,711 |
| 25 | 8,661,135 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,780,340 |
| 30 | 8,679,785 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,141,137 |
| 35 | 8,026,311 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6,923,239 |
| 40 | 6,796,256 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5,872,249 |
| 45 | 5,725,242 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.97 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4,894,477 |
| 50 | 4,724,242 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.97 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,873,065 |
| 55 | 3,684,386 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.95 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,251,658 |
| 60 | 3,028,492 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.93 | 0 | 0 | 0 | 0 | 0 | 0 | 2,637,664 |
| 65 | 2,378,899 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.90 | 0 | 0 | 0 | 0 | 0 | 2,157,487 |
| 70 | 1,843,919 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.85 | 0 | 0 | 0 | 0 | 1,545,333 |
| 75 | 1,209,249 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.78 | 0 | 0 | 0 | 949,973 |
| 80+ | 697,882 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.44 | 0.44 | 0 | 647,676 |

Matrix including fertility and (mortality+ net mobility)

| age | w^{t+10} MIDDLE Middle ²⁰⁰⁰ | $R_{MIDDLE} = S_{MIDDLE} + M_{MIDDLE}$ | | | | | | | | | | | | | | w^{t+5} MIDDLE Middle ¹⁹⁹⁵ | | | | |
|-----|--|--|------|------|------|------|------|------|------|------|------|------|------|------|------|---|------|------|---|------------|
| 0 | 8,497,309 | 0 | 0 | 0.09 | 0.21 | 0.24 | 0.18 | 0.11 | 0.06 | 0.02 | 0.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,991,861 |
| 5 | 8,754,745 | 1.10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,513,556 |
| 10 | 10,054,293 | 0 | 1.18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 9,625,562 |
| 15 | 11,851,869 | 0 | 0 | 1.23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 10,443,057 |
| 20 | 11,529,767 | 0 | 0 | 0 | 1.10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 9,724,897 |
| 25 | 9,939,737 | 0 | 0 | 0 | 0 | 1.02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 9,060,911 |
| 30 | 9,256,250 | 0 | 0 | 0 | 0 | 0 | 1.02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8,412,995 |
| 35 | 8,817,801 | 0 | 0 | 0 | 0 | 0 | 0 | 1.05 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7,360,175 |
| 40 | 7,840,921 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.07 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6,372,693 |
| 45 | 6,617,813 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.04 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5,213,248 |
| 50 | 5,370,901 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.03 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4,133,973 |
| 55 | 4,231,306 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3,498,655 |
| 60 | 3,784,060 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.08 | 0 | 0 | 0 | 0 | 0 | 0 | 3,063,055 |
| 65 | 3,051,887 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.00 | 0 | 0 | 0 | 0 | 0 | 2,383,446 |
| 70 | 2,438,744 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.02 | 0 | 0 | 0 | 0 | 1,850,075 |
| 75 | 1,570,265 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.85 | 0 | 0 | 0 | 1,030,389 |
| 80+ | 1,536,606 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.73 | 0.73 | 0 | 1,079,725 |

Appendix A. Cont'd.

- Projection models for the Brazilian rich, 1980-1985

Matrix including fertility and mortality

| W_{Rich}^{t+5} Rich ¹⁹⁸⁵ | S_{Rich} | | | | | | | | | | | | | | | W_{Rich}^t Rich ¹⁹⁸⁰ | | | |
|--|------------|-------|-------|-------|-------|------|------|------|------|------|------|------|------|------|------|--------------------------------------|------|---|---------|
| 61,924 | 0 | 0 | 0.02 | 0.08 | 0.13 | 0.12 | 0.08 | 0.04 | 0.02 | 0.01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 66,325 |
| 66,038 | 0.996 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 56,725 |
| 56,644 | 0 | 0.999 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 63,500 |
| 63,372 | 0 | 0 | 0.998 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 105,780 |
| 105,430 | 0 | 0 | 0 | 0.997 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 131,925 |
| 131,364 | 0 | 0 | 0 | 0 | 0.996 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 146,405 |
| 145,673 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 130,930 |
| 130,134 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 96,195 |
| 95,470 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 82,425 |
| 81,637 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 83,005 |
| 81,990 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 95,080 |
| 93,585 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 84,010 |
| 82,272 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 62,470 |
| 60,730 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.97 | 0 | 0 | 0 | 0 | 0 | 49,065 |
| 47,021 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.96 | 0 | 0 | 0 | 0 | 29,060 |
| 27,154 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.93 | 0 | 0 | 0 | 17,850 |
| 14,719 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.48 | 0.48 | 0 | 12,685 |

Matrix including fertility and (mortality+ net mobility)

| W_{Rich}^{t+5} Rich ¹⁹⁸⁵ | $R_{Rich} = S_{Rich} + M_{Rich}$ | | | | | | | | | | | | | | | W_{Rich}^t Rich ¹⁹⁸⁰ | | | |
|--|----------------------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|--------------------------------------|------|---|---------|
| 76,890 | 0 | 0 | 0.04 | 0.15 | 0.15 | 0.14 | 0.08 | 0.05 | 0.02 | 0.01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 66,325 |
| 71,611 | 1.08 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 56,725 |
| 80,278 | 0 | 1.42 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 63,500 |
| 112,744 | 0 | 0 | 1.78 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 105,780 |
| 201,497 | 0 | 0 | 0 | 1.90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 131,925 |
| 154,990 | 0 | 0 | 0 | 0 | 1.17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 146,405 |
| 185,101 | 0 | 0 | 0 | 0 | 0 | 1.26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 130,930 |
| 116,765 | 0 | 0 | 0 | 0 | 0 | 0 | 0.89 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 96,195 |
| 116,271 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 82,425 |
| 87,536 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.06 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 83,005 |
| 108,997 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 95,080 |
| 92,717 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 84,010 |
| 89,874 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.07 | 0 | 0 | 0 | 0 | 0 | 0 | 62,470 |
| 50,396 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.81 | 0 | 0 | 0 | 0 | 0 | 49,065 |
| 50,110 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.02 | 0 | 0 | 0 | 0 | 29,060 |
| 22,619 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.78 | 0 | 0 | 0 | 17,850 |
| 25,947 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.85 | 0.85 | 0 | 12,685 |

Appendix A. Cont'd.

- Projection models for the Brazilian rich, 1985-1990

Matrix including fertility and mortality

| age | w^{t+10}_{RICH} Rich ¹⁹⁹⁰ * | S_{RICH} | | | | | | | | | | | | | | w^{t+5}_{RICH} Rich ¹⁹⁸⁵ | | | |
|-----|---|------------|-------|-------|-------|-------|------|------|------|------|------|------|------|------|------|--|------|------|---------|
| 0 | 50,050 | 0 | 0 | 0.02 | 0.08 | 0.12 | 0.11 | 0.07 | 0.04 | 0.01 | 0.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 61,924 |
| 5 | 61,657 | 0.996 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 66,038 |
| 10 | 65,944 | 0 | 0.999 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 56,644 |
| 15 | 56,530 | 0 | 0 | 0.998 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 63,372 |
| 20 | 63,162 | 0 | 0 | 0 | 0.997 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 105,430 |
| 25 | 104,981 | 0 | 0 | 0 | 0 | 0.996 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 131,364 |
| 30 | 130,707 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 145,673 |
| 35 | 144,787 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 130,134 |
| 40 | 129,153 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 95,470 |
| 45 | 94,557 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 81,637 |
| 50 | 80,639 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 81,990 |
| 55 | 80,700 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 93,585 |
| 60 | 91,648 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 82,272 |
| 65 | 79,980 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.97 | 0 | 0 | 0 | 0 | 60,730 |
| 70 | 58,201 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.96 | 0 | 0 | 0 | 47,021 |
| 75 | 43,937 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.93 | 0 | 0 | 27,154 |
| 80+ | 20,184 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.48 | 0.48 | 14,719 |

Matrix including fertility and (mortality+ net mobility)

| age | w^{t+10}_{RICH} Rich ¹⁹⁹⁰ | $R_{RICH} = S_{RICH} + M_{RICH}$ | | | | | | | | | | | | | | w^{t+5}_{RICH} Rich ¹⁹⁸⁵ | | | |
|-----|---|----------------------------------|-------|-------|-------|-------|------|------|------|------|------|------|------|------|------|--|------|------|---------|
| 0 | 86,727 | 0 | 0 | 0.04 | 0.13 | 0.14 | 0.13 | 0.08 | 0.04 | 0.02 | 0.01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 76,890 |
| 5 | 83,018 | 1.080 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 71,611 |
| 10 | 101,344 | 0 | 1.415 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 80,278 |
| 15 | 142,532 | 0 | 0 | 1.775 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 112,744 |
| 20 | 214,762 | 0 | 0 | 0 | 1.905 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 201,497 |
| 25 | 236,726 | 0 | 0 | 0 | 0 | 1.175 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 154,990 |
| 30 | 195,955 | 0 | 0 | 0 | 0 | 0 | 1.26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 185,101 |
| 35 | 165,075 | 0 | 0 | 0 | 0 | 0 | 0 | 0.89 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 116,765 |
| 40 | 141,134 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 116,271 |
| 45 | 123,481 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.06 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 87,536 |
| 50 | 114,947 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 108,997 |
| 55 | 106,288 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 92,717 |
| 60 | 99,189 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.07 | 0 | 0 | 0 | 0 | 0 | 89,874 |
| 65 | 72,504 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.81 | 0 | 0 | 0 | 0 | 50,396 |
| 70 | 51,469 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.02 | 0 | 0 | 0 | 50,110 |
| 75 | 39,003 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.78 | 0 | 0 | 22,619 |
| 80+ | 41,270 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.85 | 0.85 | 25,947 |

Appendix A. Cont'd.

- Projection models for the Brazilian rich, 1990-1995

Matrix including fertility and mortality

| W_{Rich}^{t+5} Rich ¹⁹⁹⁵ | S_{Rich} | | | | | | | | | | | | | | | W_{Rich}^t Rich ¹⁹⁹⁰ | | |
|--|------------|-------|-------|-------|-------|------|------|------|------|------|------|------|------|------|------|--------------------------------------|------|---------|
| 72,619 | 0 | 0 | 0.03 | 0.08 | 0.10 | 0.08 | 0.05 | 0.02 | 0.01 | 0.01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 79,341 |
| 79,204 | 0.998 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 90,402 |
| 90,322 | 0 | 0.999 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 101,344 |
| 101,186 | 0 | 0 | 0.998 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 142,532 |
| 142,144 | 0 | 0 | 0 | 0.997 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 214,762 |
| 214,023 | 0 | 0 | 0 | 0 | 0.997 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 236,726 |
| 235,802 | 0 | 0 | 0 | 0 | 0 | 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 195,955 |
| 195,064 | 0 | 0 | 0 | 0 | 0 | 0 | 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 165,075 |
| 164,153 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 141,134 |
| 140,122 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 123,481 |
| 122,326 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 114,947 |
| 113,532 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 106,288 |
| 104,544 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 99,189 |
| 96,960 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 72,504 |
| 70,126 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.97 | 0 | 0 | 0 | 51,469 |
| 48,849 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.95 | 0 | 0 | 39,003 |
| 39,053 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.49 | 0.49 | 41,270 |

Matrix including fertility and (mortality+ net mobility)

| W_{Rich}^{t+5} Rich ¹⁹⁹⁵ | $R_{Rich} = S_{Rich} + M_{Rich}$ | | | | | | | | | | | | | | | W_{Rich}^t Rich ¹⁹⁹⁰ | | |
|--|----------------------------------|-------|-------|-------|-------|------|------|------|------|------|------|------|------|------|------|--------------------------------------|------|---------|
| 88,537 | 0 | 0 | 0.05 | 0.11 | 0.12 | 0.09 | 0.06 | 0.03 | 0.02 | 0.01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 79,341 |
| 94,309 | 1.189 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 90,402 |
| 121,381 | 0 | 1.343 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 101,344 |
| 150,639 | 0 | 0 | 1.486 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 142,532 |
| 207,968 | 0 | 0 | 0 | 1.459 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 214,762 |
| 250,330 | 0 | 0 | 0 | 0 | 1.166 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 236,726 |
| 233,161 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 195,955 |
| 212,701 | 0 | 0 | 0 | 0 | 0 | 0 | 1.09 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 165,075 |
| 186,316 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 141,134 |
| 180,213 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 123,481 |
| 156,120 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 114,947 |
| 138,835 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.21 | 0 | 0 | 0 | 0 | 0 | 0 | 106,288 |
| 113,491 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.07 | 0 | 0 | 0 | 0 | 0 | 99,189 |
| 97,937 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 72,504 |
| 65,518 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.90 | 0 | 0 | 0 | 51,469 |
| 43,383 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.84 | 0 | 0 | 39,003 |
| 46,340 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.58 | 0.58 | 41,270 |

Appendix A. Cont'd.

- Projection models for the Brazilian rich, 1995-2000

Matrix including fertility and mortality

| age | $w^{t+10}_{RICH} \cdot Rich^{2000}$ | S_{RICH} | | | | | | | | | | | | | | $w^{t+5}_{RICH} \cdot Rich^{1995}$ | | | |
|-----|-------------------------------------|------------|-------|-------|-------|-------|------|------|------|------|------|------|------|------|------|------------------------------------|------|------|---------|
| 0 | 53,423 | 0 | 0 | 0.02 | 0.07 | 0.08 | 0.07 | 0.04 | 0.02 | 0.01 | 0.01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 72,619 |
| 5 | 72,493 | 0.998 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 79,204 |
| 10 | 79,134 | 0 | 0.999 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 90,322 |
| 15 | 90,182 | 0 | 0 | 0.998 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 101,186 |
| 20 | 100,911 | 0 | 0 | 0 | 0.997 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 142,144 |
| 25 | 141,655 | 0 | 0 | 0 | 0 | 0.997 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 214,023 |
| 30 | 213,188 | 0 | 0 | 0 | 0 | 0 | 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 235,802 |
| 35 | 234,729 | 0 | 0 | 0 | 0 | 0 | 0 | 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 195,064 |
| 40 | 193,975 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 164,153 |
| 45 | 162,976 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 140,122 |
| 50 | 138,811 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 122,326 |
| 55 | 120,821 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 0 | 0 | 113,532 |
| 60 | 111,669 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 104,544 |
| 65 | 102,195 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 96,960 |
| 70 | 93,780 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.97 | 0 | 0 | 0 | 70,126 |
| 75 | 66,557 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.95 | 0 | 0 | 48,849 |
| 80+ | 42,765 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.49 | 0.49 | 39,053 |

Matrix including fertility and (mortality+ net mobility)

| age | $w^{t+10}_{RICH} \cdot Rich^{2000}$ | $R_{RICH} = S_{RICH} + M_{RICH}$ | | | | | | | | | | | | | | $w^{t+5}_{RICH} \cdot Rich^{1995}$ | | | |
|-----|-------------------------------------|----------------------------------|-------|-------|-------|-------|------|------|------|------|------|------|------|------|------|------------------------------------|------|------|---------|
| 0 | 80,898 | 0 | 0 | 0.04 | 0.09 | 0.10 | 0.08 | 0.05 | 0.02 | 0.02 | 0.01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 88,537 |
| 5 | 105,240 | 1.189 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 94,309 |
| 10 | 126,627 | 0 | 1.343 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 121,381 |
| 15 | 180,422 | 0 | 0 | 1.486 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 150,639 |
| 20 | 219,796 | 0 | 0 | 0 | 1.459 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 207,968 |
| 25 | 242,410 | 0 | 0 | 0 | 0 | 1.166 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 250,330 |
| 30 | 246,560 | 0 | 0 | 0 | 0 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 233,161 |
| 35 | 253,086 | 0 | 0 | 0 | 0 | 0 | 0 | 1.09 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 212,701 |
| 40 | 240,071 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 186,316 |
| 45 | 237,906 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 180,213 |
| 50 | 227,848 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 156,120 |
| 55 | 188,565 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.21 | 0 | 0 | 0 | 0 | 0 | 0 | 138,835 |
| 60 | 148,243 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.07 | 0 | 0 | 0 | 0 | 0 | 113,491 |
| 65 | 112,058 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.99 | 0 | 0 | 0 | 0 | 97,937 |
| 70 | 88,500 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.90 | 0 | 0 | 0 | 65,518 |
| 75 | 55,225 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.84 | 0 | 0 | 43,383 |
| 80+ | 51,795 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.58 | 0.58 | 46,340 |

Appendix B. Life-tables reflecting the combined effect of mortality and net mobility

Brazilian **poor** class, 1990-2000: $l_0= 1,647,279$ and $L_0= 8,016,094$

| | ${}_nN_x$ (1990) | ${}_nN_x$ (2000) | ${}_nN_x^*$ | ${}_nP_x^*$ | ${}_nL_x^{NEW}$ | I_x^* | T_x^* | e_x^* | Proj. 1995* | Proj 2000* |
|----|------------------|------------------|-------------|--------------|-----------------|-----------|------------|---------|-------------|------------|
| | (1) | (2) | (3) | (13) | (14) | (15) | (16) | (17) | (18) | (19) |
| 0 | 8,663,037 | 7,811,367 | 8,226,187 | | 8,016,094 | 1,647,279 | 45,858,623 | 27.84 | | |
| 5 | 8,938,082 | 7,558,446 | 8,219,368 | 0.949 | 7,605,558 | 1,562,165 | 37,842,528 | 24.22 | 8,219,368 | |
| 10 | 8,190,257 | 7,156,714 | 7,656,065 | 0.871 | 6,622,262 | 1,422,782 | 30,236,970 | 21.25 | 7,782,508 | 7,156,714 |
| 15 | 5,901,452 | 5,888,288 | 5,894,866 | 0.757 | 5,010,439 | 1,163,270 | 23,614,708 | 20.30 | 6,196,793 | 5,888,288 |
| 20 | 4,250,221 | 4,338,445 | 4,294,107 | 0.700 | 3,507,865 | 851,830 | 18,604,269 | 21.84 | 4,131,674 | 4,338,445 |
| 25 | 3,820,620 | 3,612,924 | 3,715,321 | 0.874 | 3,067,438 | 657,530 | 15,096,404 | 22.96 | 3,716,588 | 3,612,924 |
| 30 | 3,468,164 | 3,477,567 | 3,472,862 | 0.936 | 2,870,165 | 593,760 | 12,028,966 | 20.26 | 3,574,909 | 3,477,567 |
| 35 | 2,981,957 | 3,149,666 | 3,064,665 | 0.881 | 2,528,753 | 539,892 | 9,158,801 | 16.96 | 3,055,619 | 3,149,666 |
| 40 | 2,446,579 | 2,426,178 | 2,436,357 | 0.794 | 2,007,844 | 453,660 | 6,630,049 | 14.61 | 2,367,690 | 2,426,178 |
| 45 | 1,872,958 | 1,854,287 | 1,863,599 | 0.783 | 1,572,468 | 358,031 | 4,622,205 | 12.91 | 1,916,069 | 1,854,287 |
| 50 | 1,511,877 | 1,448,374 | 1,479,785 | 0.756 | 1,188,643 | 276,111 | 3,049,737 | 11.05 | 1,415,786 | 1,448,374 |
| 55 | 1,189,421 | 1,031,336 | 1,107,562 | 0.728 | 865,873 | 205,452 | 1,861,094 | 9.06 | 1,101,334 | 1,031,336 |
| 60 | 1,005,328 | 666,541 | 818,592 | 0.605 | 524,037 | 138,991 | 995,221 | 7.16 | 719,853 | 666,541 |
| 65 | 815,921 | 383,937 | 559,698 | 0.533 | 279,498 | 80,353 | 471,184 | 5.86 | 536,197 | 383,937 |
| 70 | 560,997 | 224,738 | 355,074 | 0.419 | 117,147 | 39,664 | 191,687 | 4.83 | 341,980 | 224,738 |
| 75 | 381,727 | 142,425 | 233,168 | 0.416 | 48,788 | 16,594 | 74,540 | 4.49 | 233,640 | 142,425 |
| 80 | 327,696 | 165,387 | 232,802 | 0.345 | 25,752 | 7,454 | 25,752 | 3.45 | 245,087 | 165,387 |

Appendix B. Cont'd.

Brazilian **middle** class, 1980-1990: $l_0= 1,930,080$ and $L_0= 8,020,094$

| | nN_x (1980) | nN_x (1990) | nN_x^* | nPx^* | nL_x^{NEW} | I_x^* | T_x^* | e_x^* | Proj. 1985* | Proj 1990* |
|----|---------------|---------------|-----------|--------------|--------------|-----------|------------|---------|-------------|------------|
| | (1) | (2) | (3) | (13) | (14) | (15) | (16) | (17) | (18) | (19) |
| 0 | 8,813,965 | 7,771,690 | 8,276,437 | | 8,020,094 | 1,930,080 | 98,237,854 | 50.90 | | |
| 5 | 7,595,495 | 8,150,525 | 7,868,118 | 0.893 | 7,159,439 | 1,517,953 | 90,217,760 | 59.43 | 7,868,118 | |
| 10 | 8,091,295 | 8,481,387 | 8,284,045 | 1.078 | 7,717,471 | 1,487,691 | 83,058,322 | 55.83 | 8,187,515 | 8,481,387 |
| 15 | 9,403,650 | 8,808,301 | 9,101,109 | 1.076 | 8,302,617 | 1,602,009 | 75,340,851 | 47.03 | 8,704,785 | 8,808,301 |
| 20 | 8,797,045 | 8,865,066 | 8,830,990 | 1.018 | 8,455,493 | 1,675,811 | 67,038,234 | 40.00 | 9,576,799 | 8,865,066 |
| 25 | 6,988,560 | 8,235,452 | 7,586,432 | 0.860 | 7,271,199 | 1,572,669 | 58,582,741 | 37.25 | 7,564,913 | 8,235,452 |
| 30 | 5,314,115 | 7,022,285 | 6,108,783 | 0.928 | 6,749,640 | 1,402,084 | 51,311,542 | 36.60 | 6,487,274 | 7,022,285 |
| 35 | 4,240,290 | 5,981,968 | 5,036,396 | 0.922 | 6,223,898 | 1,297,354 | 44,561,902 | 34.35 | 4,900,189 | 5,981,968 |
| 40 | 3,938,250 | 5,020,153 | 4,446,416 | 1.024 | 6,376,268 | 1,260,017 | 38,338,005 | 30.43 | 4,344,099 | 5,020,153 |
| 45 | 3,392,195 | 4,012,628 | 3,689,393 | 0.924 | 5,889,736 | 1,226,600 | 31,961,737 | 26.06 | 3,637,747 | 4,012,628 |
| 50 | 3,053,805 | 3,418,176 | 3,230,858 | 0.940 | 5,534,236 | 1,142,397 | 26,072,001 | 22.82 | 3,187,444 | 3,418,176 |
| 55 | 2,346,940 | 2,832,030 | 2,578,101 | 0.888 | 4,917,144 | 1,045,138 | 20,537,765 | 19.65 | 2,713,292 | 2,832,030 |
| 60 | 1,808,335 | 2,392,168 | 2,079,866 | 0.882 | 4,335,189 | 925,233 | 15,620,622 | 16.88 | 2,069,175 | 2,392,168 |
| 65 | 1,464,795 | 1,808,125 | 1,627,431 | 0.874 | 3,788,256 | 812,345 | 11,285,433 | 13.89 | 1,580,193 | 1,808,125 |
| 70 | 947,850 | 1,213,997 | 1,072,701 | 0.768 | 2,910,360 | 669,862 | 7,497,177 | 11.19 | 1,125,341 | 1,213,997 |
| 75 | 601,880 | 804,328 | 695,779 | 0.715 | 2,080,155 | 499,052 | 4,586,816 | 9.19 | 677,468 | 804,328 |
| 80 | 429,925 | 678,384 | 540,050 | 0.546 | 2,506,662 | 458,682 | 2,506,662 | 5.46 | 563,874 | 678,384 |

Brazilian **middle** class, 1990-2000: $l_0= 1,709,937$ and $L_0= 8,295,366$

| | nN_x (1990) | nN_x (2000) | nN_x^* | nPx^* | nL_x^{NEW} | I_x^* | T_x^* | e_x^* | Proj. 1995* | Proj 2000* |
|----|---------------|---------------|------------|--------------|--------------|-----------|-------------|---------|-------------|------------|
| | (1) | (2) | (3) | (13) | (14) | (15) | (16) | (17) | (18) | (19) |
| 0 | 7,771,690 | 8,477,652 | 8,117,000 | | 8,295,366 | 1,709,937 | 298,406,941 | 174.51 | | |
| 5 | 8,150,525 | 8,892,758 | 8,513,557 | 1.095 | 9,087,222 | 1,738,259 | 290,111,575 | 166.90 | 8,513,557 | |
| 10 | 8,481,387 | 10,054,293 | 9,234,411 | 1.181 | 10,731,776 | 1,981,900 | 281,024,353 | 141.80 | 9,625,562 | 10,054,293 |
| 15 | 8,808,301 | 11,851,870 | 10,217,379 | 1.231 | 13,213,942 | 2,394,572 | 270,292,577 | 112.88 | 10,443,057 | 11,851,870 |
| 20 | 8,865,066 | 11,529,766 | 10,110,002 | 1.104 | 14,588,991 | 2,780,293 | 257,078,635 | 92.46 | 9,724,897 | 11,529,766 |
| 25 | 8,235,452 | 9,939,737 | 9,047,554 | 1.022 | 14,911,288 | 2,950,028 | 242,489,645 | 82.20 | 9,060,911 | 9,939,737 |
| 30 | 7,022,285 | 9,256,250 | 8,062,259 | 1.022 | 15,232,751 | 3,014,404 | 227,578,357 | 75.50 | 8,412,996 | 9,256,250 |
| 35 | 5,981,968 | 8,817,802 | 7,262,769 | 1.048 | 15,965,702 | 3,119,845 | 212,345,606 | 68.06 | 7,360,175 | 8,817,802 |
| 40 | 5,020,153 | 7,840,921 | 6,273,964 | 1.065 | 17,008,537 | 3,297,424 | 196,379,904 | 59.56 | 6,372,694 | 7,840,921 |
| 45 | 4,012,628 | 6,617,813 | 5,153,137 | 1.038 | 17,662,753 | 3,467,129 | 179,371,367 | 51.73 | 5,213,248 | 6,617,813 |
| 50 | 3,418,176 | 5,370,901 | 4,284,703 | 1.030 | 18,196,890 | 3,585,964 | 161,708,614 | 45.09 | 4,133,974 | 5,370,901 |
| 55 | 2,832,030 | 4,231,306 | 3,461,674 | 1.024 | 18,625,327 | 3,682,222 | 143,511,724 | 38.97 | 3,498,655 | 4,231,306 |
| 60 | 2,392,168 | 3,784,060 | 3,008,672 | 1.082 | 20,144,701 | 3,877,003 | 124,886,398 | 32.21 | 3,063,055 | 3,784,060 |
| 65 | 1,808,125 | 3,051,887 | 2,349,083 | 0.996 | 20,071,252 | 4,021,595 | 104,741,697 | 26.04 | 2,383,446 | 3,051,887 |
| 70 | 1,213,997 | 2,438,744 | 1,720,648 | 1.023 | 20,536,921 | 4,060,817 | 84,670,445 | 20.85 | 1,850,075 | 2,438,744 |
| 75 | 804,328 | 1,570,265 | 1,123,836 | 0.849 | 17,430,867 | 3,796,779 | 64,133,523 | 16.89 | 1,030,389 | 1,570,265 |
| 80 | 678,384 | 1,536,606 | 1,020,984 | 0.728 | 46,702,656 | 6,413,352 | 46,702,656 | 7.28 | 1,079,725 | 1,536,606 |

Appendix B. Cont'd.

Brazilian rich class, 1980-1990: $l_0= 15,120$ and $L_0= 75,865$

| | nN_x (1980) | nN_x (1990) | nN_x^* | nP_x^* | nL_x^{NEW} | l_x^* | T_x^* | e_x^* | Proj. 1985* | Proj 1990* |
|----|---------------|---------------|----------|--------------|--------------|---------|------------|---------|-------------|------------|
| | (1) | (2) | (3) | (13) | (14) | (15) | (16) | (17) | (18) | (19) |
| 0 | 66,325 | 79,341 | 72,542 | | 75,865 | 15,120 | 11,757,588 | 777.61 | | |
| 5 | 56,725 | 90,402 | 71,611 | 1.080 | 81,911 | 15,778 | 11,681,723 | 740.40 | 71,611 | |
| 10 | 63,500 | 101,344 | 80,220 | 1.415 | 115,921 | 19,783 | 11,599,812 | 586.35 | 80,278 | 101,344 |
| 15 | 105,780 | 142,532 | 122,789 | 1.775 | 205,816 | 32,174 | 11,483,892 | 356.93 | 112,744 | 142,532 |
| 20 | 131,925 | 214,762 | 168,323 | 1.905 | 392,053 | 59,787 | 11,278,076 | 188.64 | 201,497 | 214,762 |
| 25 | 146,405 | 236,726 | 186,166 | 1.175 | 460,597 | 85,265 | 10,886,023 | 127.67 | 154,990 | 236,726 |
| 30 | 130,930 | 195,955 | 160,176 | 1.264 | 582,337 | 104,293 | 10,425,426 | 99.96 | 185,101 | 195,955 |
| 35 | 96,195 | 165,075 | 126,014 | 0.892 | 519,335 | 110,167 | 9,843,089 | 89.35 | 116,765 | 165,075 |
| 40 | 82,425 | 141,134 | 107,856 | 1.209 | 627,721 | 114,706 | 9,323,755 | 81.28 | 116,271 | 141,134 |
| 45 | 83,005 | 123,481 | 101,240 | 1.062 | 666,643 | 129,436 | 8,696,034 | 67.18 | 87,536 | 123,481 |
| 50 | 95,080 | 114,947 | 104,542 | 1.313 | 875,395 | 154,204 | 8,029,390 | 52.07 | 108,997 | 114,947 |
| 55 | 84,010 | 106,288 | 94,495 | 0.975 | 853,636 | 172,903 | 7,153,995 | 41.38 | 92,717 | 106,288 |
| 60 | 62,470 | 99,189 | 78,717 | 1.070 | 913,224 | 176,686 | 6,300,360 | 35.66 | 89,874 | 99,189 |
| 65 | 49,065 | 72,504 | 59,644 | 0.807 | 736,720 | 164,994 | 5,387,135 | 32.65 | 50,396 | 72,504 |
| 70 | 29,060 | 51,469 | 38,674 | 1.021 | 752,412 | 148,913 | 4,650,415 | 31.23 | 50,110 | 51,469 |
| 75 | 17,850 | 39,003 | 26,386 | 0.778 | 585,634 | 133,805 | 3,898,003 | 29.13 | 22,619 | 39,003 |
| 80 | 12,685 | 41,270 | 22,880 | 0.850 | 3,312,369 | 389,800 | 3,312,369 | 8.50 | 25,947 | 41,270 |

Brazilian rich class, 1990-2000: $l_0= 16,685$ and $L_0= 89,879$

| | nN_x (1990) | nN_x (2000) | nN_x^* | nP_x^* | nL_x^{NEW} | l_x^* | T_x^* | e_x^* | Proj. 1995* | Proj 2000* |
|----|---------------|---------------|----------|--------------|--------------|---------|-----------|---------|-------------|------------|
| | (1) | (2) | (3) | (13) | (14) | (15) | (16) | (17) | (18) | (19) |
| 0 | 79,341 | 93,694 | 86,219 | | 89,879 | 16,685 | 8,772,364 | 525.78 | | |
| 5 | 90,402 | 98,384 | 94,309 | 1.189 | 106,835 | 19,671 | 8,682,485 | 441.38 | 94,309 | |
| 10 | 101,344 | 126,627 | 113,282 | 1.343 | 143,446 | 25,028 | 8,575,650 | 342.64 | 121,382 | 126,627 |
| 15 | 142,532 | 180,423 | 160,362 | 1.486 | 213,219 | 35,666 | 8,432,204 | 236.42 | 150,638 | 180,423 |
| 20 | 214,762 | 219,795 | 217,264 | 1.459 | 311,107 | 52,433 | 8,218,985 | 156.75 | 207,968 | 219,795 |
| 25 | 236,726 | 242,410 | 239,551 | 1.166 | 362,631 | 67,374 | 7,907,878 | 117.37 | 250,330 | 242,410 |
| 30 | 195,955 | 246,560 | 219,806 | 0.985 | 357,169 | 71,980 | 7,545,247 | 104.82 | 233,161 | 246,560 |
| 35 | 165,075 | 253,086 | 204,397 | 1.085 | 387,692 | 74,486 | 7,188,078 | 96.50 | 212,701 | 253,086 |
| 40 | 141,134 | 240,071 | 184,071 | 1.129 | 437,579 | 82,527 | 6,800,386 | 82.40 | 186,317 | 240,071 |
| 45 | 123,481 | 237,907 | 171,397 | 1.277 | 558,743 | 99,632 | 6,362,806 | 63.86 | 180,213 | 237,907 |
| 50 | 114,947 | 227,848 | 161,834 | 1.264 | 706,433 | 126,518 | 5,804,063 | 45.88 | 156,120 | 227,848 |
| 55 | 106,288 | 188,564 | 141,570 | 1.208 | 853,241 | 155,967 | 5,097,630 | 32.68 | 138,834 | 188,564 |
| 60 | 99,189 | 148,243 | 121,260 | 1.068 | 911,064 | 176,431 | 4,244,389 | 24.06 | 113,491 | 148,243 |
| 65 | 72,504 | 112,058 | 90,137 | 0.987 | 899,563 | 181,063 | 3,333,325 | 18.41 | 97,937 | 112,058 |
| 70 | 51,469 | 88,500 | 67,491 | 0.904 | 812,886 | 171,245 | 2,433,763 | 14.21 | 65,518 | 88,500 |
| 75 | 39,003 | 55,225 | 46,410 | 0.843 | 685,183 | 149,807 | 1,620,877 | 10.82 | 43,384 | 55,225 |
| 80 | 41,270 | 51,795 | 46,234 | 0.577 | 935,694 | 162,088 | 935,694 | 5.77 | 46,339 | 51,795 |

Appendix B. Cont'd.

Brazilian **poor+ middle+ rich** classes, 1980-1990: $l_0= 3,675,311$ and $L_0= 16,477,038$

| | nN_x (1980) | nN_x (1990) | nN_x^* | nP_x^* | nL_x^{NEW} | l_x^* | T_x^* | e_x^* | Proj. 1985* | Proj 1990* |
|----|---------------|---------------|------------|--------------|--------------|-----------|-------------|---------|-------------|------------|
| | (1) | (2) | (3) | (13) | (14) | (15) | (16) | (17) | (18) | (19) |
| 0 | 16,366,445 | 16,514,068 | 16,440,091 | | 16,477,038 | 3,675,311 | 225,603,621 | 61.38 | | |
| 5 | 14,727,495 | 17,179,010 | 15,906,093 | 0.972 | 16,013,575 | 3,249,061 | 209,126,584 | 64.37 | 15,906,093 | |
| 10 | 14,206,040 | 16,772,987 | 15,436,247 | 1.055 | 16,886,327 | 3,289,990 | 193,113,009 | 58.70 | 15,530,155 | 16,772,987 |
| 15 | 13,488,845 | 14,852,285 | 14,154,157 | 0.956 | 16,149,262 | 3,303,559 | 176,226,682 | 53.34 | 13,585,966 | 14,852,285 |
| 20 | 11,325,870 | 13,330,050 | 12,287,164 | 0.981 | 15,845,061 | 3,199,432 | 160,077,421 | 50.03 | 13,234,758 | 13,330,050 |
| 25 | 9,305,880 | 12,292,799 | 10,695,574 | 0.929 | 14,717,318 | 3,056,238 | 144,232,359 | 47.19 | 10,519,772 | 12,292,799 |
| 30 | 7,592,170 | 10,686,405 | 9,007,386 | 1.016 | 14,950,440 | 2,966,776 | 129,515,041 | 43.66 | 9,453,285 | 10,686,405 |
| 35 | 6,314,405 | 9,129,001 | 7,592,378 | 0.966 | 14,437,582 | 2,938,802 | 114,564,601 | 38.98 | 7,331,729 | 9,129,001 |
| 40 | 5,698,415 | 7,607,865 | 6,584,282 | 1.038 | 14,981,347 | 2,941,893 | 100,127,019 | 34.03 | 6,552,225 | 7,607,865 |
| 45 | 4,649,135 | 6,009,067 | 5,285,543 | 0.917 | 13,739,442 | 2,872,079 | 85,145,672 | 29.65 | 5,226,035 | 6,009,067 |
| 50 | 4,080,040 | 5,044,999 | 4,536,937 | 0.965 | 13,263,493 | 2,700,293 | 71,406,230 | 26.44 | 4,488,084 | 5,044,999 |
| 55 | 3,117,940 | 4,127,739 | 3,587,484 | 0.920 | 12,198,578 | 2,546,207 | 58,142,737 | 22.84 | 3,752,457 | 4,127,739 |
| 60 | 2,415,165 | 3,496,685 | 2,906,040 | 0.932 | 11,367,108 | 2,356,569 | 45,944,159 | 19.50 | 2,905,417 | 3,496,685 |
| 65 | 2,015,890 | 2,696,550 | 2,331,512 | 0.928 | 10,549,938 | 2,191,705 | 34,577,051 | 15.78 | 2,241,541 | 2,696,550 |
| 70 | 1,309,465 | 1,826,464 | 1,546,509 | 0.815 | 8,596,352 | 1,914,629 | 24,027,113 | 12.55 | 1,642,597 | 1,826,464 |
| 75 | 820,075 | 1,225,058 | 1,002,317 | 0.746 | 6,411,204 | 1,500,756 | 15,430,761 | 10.28 | 976,606 | 1,225,058 |
| 80 | 574,600 | 1,047,350 | 775,762 | 0.585 | 9,019,556 | 1,543,076 | 9,019,556 | 5.85 | 815,213 | 1,047,350 |

Brazilian **poor+ middle+ rich** classes, 1990-2000: $l_0= 3,434,816$ and $L_0= 16,415,453$

| | nN_x (1990) | nN_x (2000) | nN_x^* | nP_x^* | nL_x^{NEW} | l_x^* | T_x^* | e_x^* | Proj. 1995* | Proj 2000* |
|----|---------------|---------------|------------|--------------|--------------|-----------|-------------|---------|-------------|------------|
| | (1) | (2) | (3) | (13) | (14) | (15) | (16) | (17) | (18) | (19) |
| 0 | 16,514,068 | 16,382,713 | 16,448,259 | | 16,415,453 | 3,434,816 | 249,811,787 | 72.73 | | |
| 5 | 17,179,010 | 16,549,588 | 16,861,362 | 1.021 | 16,760,674 | 3,317,613 | 233,396,333 | 70.35 | 16,861,362 | |
| 10 | 16,772,987 | 17,337,634 | 17,052,974 | 1.028 | 17,234,102 | 3,399,478 | 216,635,659 | 63.73 | 17,664,254 | 17,337,634 |
| 15 | 14,852,285 | 17,920,581 | 16,314,459 | 1.015 | 17,484,187 | 3,471,829 | 199,401,557 | 57.43 | 17,016,380 | 17,920,581 |
| 20 | 13,330,050 | 16,088,006 | 14,644,245 | 0.945 | 16,530,290 | 3,401,448 | 181,917,371 | 53.48 | 14,041,978 | 16,088,006 |
| 25 | 12,292,799 | 13,795,071 | 13,022,290 | 0.982 | 16,239,629 | 3,276,992 | 165,387,081 | 50.47 | 13,095,661 | 13,795,071 |
| 30 | 10,686,405 | 12,980,377 | 11,777,672 | 0.991 | 16,096,669 | 3,233,630 | 149,147,452 | 46.12 | 12,184,583 | 12,980,377 |
| 35 | 9,129,001 | 12,220,554 | 10,562,265 | 1.003 | 16,144,189 | 3,224,086 | 133,050,783 | 41.27 | 10,717,953 | 12,220,554 |
| 40 | 7,607,865 | 10,507,170 | 8,940,757 | 0.980 | 15,826,691 | 3,197,088 | 116,906,594 | 36.57 | 8,949,467 | 10,507,170 |
| 45 | 6,009,067 | 8,710,007 | 7,234,571 | 0.973 | 15,403,219 | 3,122,991 | 101,079,903 | 32.37 | 7,404,302 | 8,710,007 |
| 50 | 5,044,999 | 7,047,123 | 5,962,611 | 0.952 | 14,660,176 | 3,006,339 | 85,676,684 | 28.50 | 5,719,193 | 7,047,123 |
| 55 | 4,127,739 | 5,451,206 | 4,743,539 | 0.953 | 13,973,237 | 2,863,341 | 71,016,508 | 24.80 | 4,808,603 | 5,451,206 |
| 60 | 3,496,685 | 4,598,844 | 4,010,076 | 0.956 | 13,363,701 | 2,733,694 | 57,043,271 | 20.87 | 3,947,680 | 4,598,844 |
| 65 | 2,696,550 | 3,547,882 | 3,093,063 | 0.899 | 12,010,302 | 2,537,400 | 43,679,570 | 17.21 | 3,142,561 | 3,547,882 |
| 70 | 1,826,464 | 2,751,982 | 2,241,962 | 0.876 | 10,517,580 | 2,252,788 | 31,669,269 | 14.06 | 2,361,404 | 2,751,982 |
| 75 | 1,225,058 | 1,767,915 | 1,471,665 | 0.749 | 7,874,207 | 1,839,179 | 21,151,689 | 11.50 | 1,367,420 | 1,767,915 |
| 80 | 1,047,350 | 1,753,788 | 1,355,297 | 0.628 | 13,277,482 | 2,115,169 | 13,277,482 | 6.28 | 1,426,451 | 1,753,788 |

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