# The crossover between life expectancies at birth and at age one: the imbalance of the life table. 

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#### Abstract

. The single most used demographic measure is the life expectancy at birth, but life expectancies at ages other than zero are also used in the study of human longevity. Our intuition tells us that the longest life expectancy is that of a new born. However, historically, the expectation of life at age one $\left(e_{1}\right)$ has exceeded the expectation of life at birth $\left(e_{0}\right)$. The crossover between $e_{0}$ and $e_{1}$ only occurred in the second half of the twentieth century in the developed world. Life tables for populations that have not achieved this crossing between life expectancy at birth and at age one are referred to here as imbalanced. The timing of this crossover is when infant mortality is equal to the inverse of life expectancy at age one. This simple relation between mortality at age zero and mortality after age one divides the world into countries that have achieved the crossover in life expectancies and those that have not. It is a within population comparison of mortality at infancy and after age one. However, results of these within comparison can be used for comparison between populations. For countries that have already achieved this crossing in life expectancies, the sex differential in the timing of the crossing is marked--females attain the crossing before males for every single population and in some cases by up to 20 years earlier. However, for most countries life expectancy at age one is still higher than life expectancy at birth and in some cases by several years. Subpopulation comparisons for the US show how black Americans are near to transitioning out of the imbalanced life table situation.


## Introduction

The life table measure most widely used to describe the mortality situation of a population is life expectancy at birth, $e_{0}$. Period life expectancy at birth is defined as the average number of years that a newborn would live given the set of death rates observed in that year (Preston et al. 2001). Changes in mortality in the first year of life strongly affect life expectancy at birth. Thus it has been suggested that instead of $e_{0}$ the study of infant mortality in conjunction with life expectancy at age one could be used as comparative measures of the level of mortality of a population (Kintner 2004). Infant mortality levels can be compared over time, or over subpopulations defined in terms of sex, race, nationality, education, social class, etc. (Frisbie et al. 2004, Martin et al. 2005, Hummer et al. 2007). An alternative comparison is to observe the relation between a level of infant mortality in a population and its mortality at adult ages (Finch and Crimmins 2004, Galobardes et al. 2004). In this study we address this alternative comparison by analyzing the imbalance in the life table.

Life expectancy is thought to be a monotonic decreasing function of age. However, this has not always been the case. Figure 1 presents life expectancy by age at different times for the population of Iceland. In 1860 life expectancy at birth was only higher than life expectancies after age 41 . Over time this changed and by 1950 the only remaining life expectancy higher than $e_{0}$ was $e_{1}$. The country comparison of the timing of the crossing between $e_{0}$ and $e_{1}$, and the relations of the life table functions at the time of this crossing are the focus of the current paper.
[Figure 1 about here]

Let an imbalance in a life table be defined as a situation where levels of life expectancy at an early age $x$ are lower than at an older age $y, e_{x}<e_{y}$. Of particular interest is the case
of an imbalance between life expectancy at birth and at age one, with the latter having higher values, $e_{0}<e_{1}$. The mathematical relations of the life table allow us to find the precise moment when this imbalance stops existing which relates life expectancies at birth, at age one, and infant mortality.

This project furthers our knowledge on life tables by showing the possibility of analyzing together levels of infant mortality with corresponding mortality at other ages. The crossover between the life expectancies can help us separate the world into countries that have passed this threshold point and those where the life table imbalance persists. The latter countries are still trying to break the paradoxical situation where a person born this year has a shorter life expectancy than a person of age one. Assuming declines in mortality over time, this can be thought as a situation where persons attaining age one obtain extra years of life expectancy, on top of the year already lived.

This imbalance in the life table can also be used as an alternative examination of the black-white life expectancy gap (Kochanek et al. 1994, Harper et al. 2007). The disadvantaged survivorship of the American black population respect to the white population (Haines 2003, Klein 2004) is well known. Although the life expectancies for each sex/race are at different levels it can be asked if each of these life tables by sex/race has reached a balanced situation between its life expectancy function. These questions are addressed in the current study.

This paper has four sections, with the present introduction as the first. Methods and data used in the project are presented in section two. The third section contains the results and it is divided in three subsections: i) the crossover in life expectancies in industrialized countries, ii) the current situation of the countries of the world and iii) the comparison of the black and white life tables in the United States. Finally the discussion is the fourth section.

## Methods

The life table is the simplest mathematical demography theory of the age structure of a population which is subjected to certain patterns of fertility and mortality interrelated by a set of mathematical functions. These specific mathematical relations describe the likelihood of persons in a population experiencing an event. The classical use of life tables is to study mortality (although other events could be studied: first birth, first marriage, exiting the job market, and others) (Preston et al. 2001, Kintner 2004). In this section we derive some relations in the life table which are present in an imbalanced life table situation mentioned above.

Let the function describing the number of survivors at age $x$ and at time $t$ in a life table be denoted as $\ell(x, t)$. Life expectancy at age $x$ and time $t$ is calculated in terms of the survival function as:

$$
\begin{equation*}
e_{x}(t)=\frac{\int_{x}^{\infty} \ell(a, t) d a}{\ell(x, t)} \tag{1}
\end{equation*}
$$

where $\omega$ is the highest age attained by a member of the population. To simplify some of the equations presented below, let the radix of the life table be equal to one, $\ell(0, t)=1$.

Demographic measures change constantly over time and the life table functions mentioned above are no exception. Several demographic textbooks list the fact that for a long time life expectancy was lower at birth than at age one. For example, Chiang (1984: 118) mentions "As a rule, the expectation of life $e_{x}$ decreases as the age $x$ increases, with the exception of the first year of life where the reverse is true because of the high mortality during the first year." However, during the second half of the twentieth century most developed countries experienced a crossover of these two life expectancies. For example, for the United States total population the crossing of $e_{0}$ and $e_{1}$ occurs in 1979, 1977 for females and 1980 for males.

Mathematically it can be shown that the crossing occurs exactly at the time when the inverse of the age-specific death rate at age zero equals the life expectancy at age one,

$$
\begin{equation*}
e_{0}(t)=e_{1}(t) \quad \Rightarrow \quad \frac{1}{{ }_{1} m_{0}}=e_{1}(t), \tag{2}
\end{equation*}
$$

where the age-specific death rate at age zero, ${ }_{1} m_{0}$, is defined as the ratio of deaths in the first year of life, ${ }_{1} d_{0}(t)$, divided by the number of person-years lived between birth and age one, ${ }_{1} L_{0}(t)=\int_{0}^{1} \ell(a, t) d a$ (Preston et al. 2001). Hereafter in the text we refer to the age-specific death rate in the first year of life as infant mortality.

In equation (2), life expectancy at birth is equal (or it is above) life expectancy at age one when the inverse of the infant mortality is equal to (or is greater than) the average length of life after age one. This relates to the fact that mortality levels at different ages are related to each other, a fact that has been used to develop models of mortality estimation (Brass 1971, Murray et al. 2003, Wilmoth and Canudas-Romo 2006). Before the crossing in life expectancies higher infant mortality than expected -i.e. respect to the mortality level at ages older than one- is observed. In a balanced situation mortality in the first year, captured by the inverse of ${ }_{1} m_{0}$, corresponds to mortality observed after age one, as captured by life expectancy at age one. The significance of equation (2) is to show the relation between mortality in the first year of life and in the ages after one in the ageaggregated measure of life expectancy. Figure 2 presents the trend in the three components of equation (2) for the total population of Sweden from 1850 to 2000.
[Here Figure 2]

During the twentieth century the reduction in infant mortality, captured by ${ }_{1} m_{0}$, turns into an exponential increase of its inverse. In 1965 the crossing in life expectancies
occurs. Of particular interest is here the difference between these two life expectancies that initially are apart by -6.7 years in 1850 (higher $e_{1}$ than $e_{0}$ ) and the difference changes to almost 0.8 of a year by 2000 .

To further analyze the components of the change over time in the difference in life expectancies (at birth minus at age one) it is possible to calculate this difference as:

$$
\begin{equation*}
e_{0}(t)-e_{1}(t)={ }_{1} d_{0}(t)\left[\frac{1}{{ }_{1} m_{0}}-e_{1}(t)\right] . \tag{3}
\end{equation*}
$$

The difference in equation (3), is equal to zero when the condition in (2) is fulfilled. However, this difference can be of further interest if the focus is on explaining trends over time in the gap in life expectancies at birth and age one. The change over time of this difference can help disentangle the causes of the reduction or increase in the life expectancies gap. By looking at the derivative with respect to time of equation (3) it is possible to study the components of change of this difference (Vaupel and CanudasRomo 2002, 2003).

Specifically, let $\Delta e_{0-1}(t)=e_{0}(t)-e_{1}(t)$ denote the difference in life expectancies at birth and age one, and a dot on top of a variable denote the partial derivative with respect to time (see Appendix for further derivations). It should be noted that initially this difference is negative, but moves to positive values when the life table achieves a balanced situation with $\Delta e_{0-1}(t)$ becoming equal to zero.

$$
\begin{equation*}
\dot{\Delta} e_{0-1}(t)=\dot{L}_{0}-\dot{d}_{0}(t) e_{1}(t)-{ }_{1} d_{0}(t) \dot{e}_{1}(t) \tag{4}
\end{equation*}
$$

The first two terms on the right of equation (4) correspond to changes occurring in the first year of life, while the third component corresponds to changes after age one. The number of deaths before age one have declined over time and the number of person years
lived before age one have increased. As a consequence the first two components will contribute to the positive increase of the difference. Opposed to this is the overall trend of increase in life expectancy at age one. Therefore, the third component of the change in life expectancy at age one opposes the increase in $\Delta e_{0-1}$. In some exceptional periods adult mortality increased, so $e_{1}$ declined. As a consequence the third component in equation (4) also contributes positively to the increase in $\Delta e_{0-1}$.

## Data

To study the trend over time in life expectancies at birth and at age one, in the components of the difference, and in infant mortality five databases are used: the Human Mortality Database (2008), Mortality Indicators Database from the United Nations (2008), Mortality Indicators from the World Health Organization (2008), Coale and Demeny Model Life Tables (1983), and US life tables by sex and race from the Center for Disease Control (1984-2006).

The Human Mortality Database (HMD) project contains detailed time series of mortality data and life tables for populations with virtually complete registration and census data. All countries from this database are used and their annual period life expectancies, at birth and at age one, are compared. For the life tables that experience a crossing of life expectancies, $e_{0}=e_{1}$, we have also used the age-specific death rate from age 0 to 1 .

United Nations (UN) and the World Health Organization (WHO) produce mortality estimates and life tables on an annual basis for all state members. The majority of the countries of the world have incomplete data (partial counts of vital events and/or populations), and their life tables have to be constructed using a combination of direct and indirect methods (United Nations 1982, Murray et al. 2003, Wilmoth and CanudasRomo 2006). Given the different methods used by each organization, the estimates coming from UN and WHO for any given country do not always match. For the current project we have chosen the lower scenario for each country, with a minimum life expectancy and maximum infant mortality. It should be noted that the infant mortality
measures coming from UN and WHO are calculated as the ratio of deaths in the first year of life in a given year divided by the births in that year. This ratio is what is normally known as the Infant Mortality Rate, IMR, although it is not a rate in the sense of occurrence over exposure (Preston et al. 2001). In the present project we use the agespecific death rate at age one, IM, calculated as the ratio of deaths divided by the person years lived between ages zero and one. Analysis of the historical mortality data from the HMD (2008) shows that the two ratios are highly correlated ( $R^{2}=0.997$ ). Furthermore, the trends over time of the two ratios are similar given that both are influenced by deaths and births.

The model life tables published by Coale and Demeny (1983) are a system of life tables that give mortality rates by sex and age, and are defined by a small number of parameters that capture the level as well as the age pattern of mortality. There are different sets of values for men and women for each of four regional families (North, South, East, and West) representing separate hypothetical trajectories of mortality change. Within each of these families, life tables are specified for different levels of mortality. All the families and levels are used in this paper.

The Center for Disease Control (CDC) prepares U.S. life tables based on final numbers of deaths by year, and population estimates by year produced under a collaborative agreement with the U.S. Census Bureau. This data source provided life tables by sex and race which are used in the analysis for the US population.

All the above-mentioned data sources were used in subsections of the results: i) The HMD is used to study the timing of the crossing in industrialized countries; ii) The UN and WHO databases are used to find the current situation with respect to the crossing in life expectancies for the different regions of the world; similarly the trend in life expectancies of the Coale-Demeny model life tables are studied in this section; and iii) to address the gap in life table imbalance for the black and white American population the data from CDC are used.

## Results

## i) The crossover in industrialized countries

For industrialized countries Table 1 and Figure 3 show the timing of the crossing of e0 and e1. There is considerable variation in the timing of the crossover. Table 1 also shows the life expectancy and the infant mortality level achieved at this time.

## [Here Table 1 and Figure 3]

The first crossing in the world occurs in 1957 for Icelandic females, for males this is not observed until 1967 in the same country. In general the time of the male crossover is later than that of females for all analyzed countries except Slovenia. However, the variation is large with one year difference for Portugal, Russia and Spain, and 14 to 18 years difference in Latvia, Lithuania, Bulgaria and Ukraine. The crossings in life expectancies occurred in the range of infant mortality of 11.6 to 14.5 per thousand deaths for females and of 13.6 and 17.5 for males.

Figure 3 shows a clear regional clustering of the crossing of the life expectancies. The Nordic countries together with Switzerland and the Netherlands were among the first to experience a balance in their life table. In the Eastern European region in contrast, the first crossings are not observed until the 1980s. Another unexpected finding is the similarity in levels of life expectancy at which the crossing occurs: Eastern Europe at a low level, Nordic and central Europe at middle levels, and finally Southern Europe at high levels.

Applying the decomposition for the change over time in the difference between life expectancy at birth minus life expectancy at age one in equation (4) allows studying of the components of this change. The results show a predominant contribution of changes in mortality before age one. However, this is not constant over all the time periods. A few periods for some countries resulted in exceptional excess mortality at adult ages which reversed the direction of the component of change in life expectancy at age one to contribute to increase the gap between $e_{0}$ and $e_{1}$. This was particularly important in
some eastern European countries where reductions in survivorship among young adults translated into narrowing the gap. Figures $4 \mathrm{a}, 4 \mathrm{~b}$ and 4 c present the life expectancy trends and the components of change of the difference $e_{0}-e_{1}$ during each of the decades since the 1960s up to the first years of the 2000s for males from Japan, Russia and the United States. The components have been aggregated into those that correspond to changes before age one "change in $\mathrm{d}(0)$ " and those after age one "change in $e_{1}$ ". Similar trends were observed for females. Across countries the changes observed in the two components are clustered into three types: 1) a decline in the components over time, 2) fluctuations over time with an apparent declining trend but interrupted by mortality crises, and finally 3) a bell shape with a peak in one of the periods and decline thereafter. The selected countries represent these three types of change: Japan for (1), Russia for (2) and USA for (3).

## [Here Figures 4a, 4b and 4c]

In 1960 the three selected countries had male life expectancy at age one above life expectancy at birth by more than one year. As observed in Figure 4a, in the 1960s changes in mortality in the first year of life accounted for almost all the reduction in the difference between life expectancies in Japan. At the end of this decade both Russia and the USA stood ready to achieve the crossover with a difference of half a year between the two life expectancies. However, Russia as well as other east European countries entered a mortality crisis in the 1970s that delayed the crossover and kept their life table imbalanced. A different situation is observed in Figure 4b in the 1990s, when the increase in mortality at adult ages, captured by the "change in $e_{1}$ ", helps to reduce the gap in life expectancies in Russia. That is, life expectancy at age one reduced and the gap between this and life expectancy at birth also declined.

It should be noted here that all the results presented in this section correspond to period life tables. The youngest cohort life tables available in the HMD (2008) are for the cohort born in 1915 which did not have a crossover in life expectancies for any country. In other words they have not achieved a minimum of infant mortality that corresponds to the adult
mortality of life expectancy at age one. Goldstein and Wachter (2006) have noted how period measures of mortality follow similar trends as cohort measures but lagged by some years. For example, the Swedish cohort of 1915 had a life expectancy of 66.37 which was not observed until the period life table of 1939. Currently, the period life expectancy at year $t$ is approximately equal to the cohort life expectancy for persons born half a century ago, or $e_{0}^{p}(t+50) \approx e_{0}^{c}(t)$. It is then likely to observe the crossings in life expectancies, at birth and age one, earlier in the cohort life tables than what has been observed in the period life tables. To explore this will necessitate waiting until persons from the cohorts born in the first half of the twentieth century are dead.

## ii) The crossover in countries of the world

The United Nations (2008) and the World Health Organization (2008) databases can be used to present the current situation of the world with respect to the crossing in life expectancies and the imbalance in life tables. Figure 5a presents the current situation of the countries of the world with respect to their life expectancy at birth and infant mortality. Four regions of the world are included in Figure 5a as suggested by the United Nations macro regions: Africa, America, Asia/Oceania, and Europe. Also included in the Figure is the line where life expectancy at age one is equal to the inverse of infant mortality, as expressed in equation (2).
[Here Figure 5a]

Most of the European countries have achieved a balanced life table, and are located to the left of the curve of equation (2). On the other hand only the island nations of the Republics of Seychelles and Mauritius have achieved this situation in Africa. Asia and the Americas have some of their countries still experiencing the imbalance in life tables while in others life expectancy at birth has the highest value. Although this plot presents the current situation, it can not be used to extrapolate possible crossing times and levels of mortality. As shown in Figure 4b for Russia unexpected retrocessions in mortality can delay or accelerate the crossing of the two life expectancies. The countries included in the HMD have been excluded from Figure 5a, but a comparison using results from Table 1 is
instructive. The life expectancy crossing occurred between the levels of 65 to 75 years for countries included in the HMD. For countries in Figure 5a experiencing the crossing in most recent years, this occurs in some cases at levels of life expectancy higher than 75 years.

Model life tables are one of the major tools for indirect estimation of mortality in populations where complete and reliable data are not available. The Coale-Demeny (1983) system of model life tables, C-D, is the best known of these systems. In Figure 5b, the difference between life expectancy at birth and at age one for all the levels of the four families of life tables in the C-D system is plotted against their level of life expectancy.

## [Here Figure 5b]

The crossing in life expectancies occurs only in the highest levels of life expectancy for the East, West and North families and it does not occur at all in the South family of the C-D system. The original C-D system was created in 1966 based on life tables before 1960, i.e. at that time no country had achieved the crossing in life expectancies at birth and age one. Model life tables created by the UN (1982) show similar patterns as those observed in Figure 5b. Currently, the families of these collections of model life tables have been extended to higher levels of life expectancy and at those high levels all families have achieved a balanced situation.

## iii) The imbalance in the life tables for the black and white population of the USA

The black-white life expectancy gap can also be studied in terms of the imbalance in the life tables. Period life tables by race and sex for the US population since 1980 are used here to study the crossover in life expectancies. As observed in Figure 6 the trend for all the groups has been an increase in life expectancy and decline in infant mortality over time. All groups have moved from right to left in Figure 6 from high infant mortality to lower levels. However, while the white population during all this period has had a balanced life table, the black population is still transitioning to the crossover point with higher life expectancy at birth than at age one. This can be observed in Figure 6 by noting
that the white population is to the left of the line where $e_{0}=e_{1}$. The female black population experienced this crossing in 1996 and black males are close to accomplishing this. The timing of the crossing in the white population is 1974 for females and 1977 for males. Note that these years are earlier than those shown in Table 1 and Figure 3 for the US population with all races included of 1977 for females and 1980 for males.
[Here Figure 6]

The gap between the black and white life expectancies has been studied (e.g. Kochanek et al. 1994, Harper et al. 2007). The novelty of the present results is to show not only that the black population has higher levels of mortality, but also that its life table is until recently in an imbalanced situation. This imbalanced situation is due to the high levels of infant mortality still prevalent which affect life expectancy at birth but not at age one, and as a consequence $e_{0}<e_{1}$.

In the last set of results we explore changes in the gap between life expectancies at birth and at age one over the last two decades of the twentieth century and the first years of the new millennium. Figures 7a and 7 b present these changes for males of the two races (similar results were observed for the female populations but with less pronounced changes).
[Here Figure 7a and 7b]

A clear difference in trends is observed between Figure 7a for the black male population as opposed to Figures 7b for the white males. While the latter group had a declining overall pattern in both components of change, the black population had an abrupt change in the period 1985-1990. During these years the component of adult mortality change, "change in $e_{1}$ ", contributes positively to balance the life table. This paradox is due to an increase in adult mortality in this population. The opposite occurs in the periods 19952000 and 2000-2003 for black males, where the component of "change in $e_{1}$ " contributes
negatively to the change in the difference $\Delta e_{0-1}(t)$ and is the most important component of this change.

## Discussion

The motivation for this research was to find the possible relations among the functions of an imbalanced life table that take place when life expectancy at birth is lower than at age one, $e_{0}<e_{1}$. This imbalance ends when the crossover between life expectancies, $e_{0}=e_{1}$, occurs. Equation (2) is a simple and profound relation between infant mortality and life expectancy at age one and it is central in explicating the mechanisms of the crossover.

The demographic transition has been described as a period of decline from high to lower levels of fertility and mortality (Lee 2003). The mortality aspect of this transition has been characterized by, among others, the increase in life expectancy at birth. However, the causes of this increase have changed over time from a dominance of child mortality reductions to a dominance of adult mortality reductions (Canudas-Romo and Wilmoth 2007). The differential in mortality decline by age has created situations of imbalanced life tables. These imbalances occur at different mortality levels as well as at different times for each country. For countries where historical information on mortality is available it is possible to analyze these disparities over time and region. The illustration of the crossover in life expectancy observed in the HMD data suggests that socio-cultural, economic, and political factors that influence the intermediate behavioral factors that shape the mortality patterns in each country have traversed borders. In other words, the timing and mortality level at which populations progress from an imbalanced life table to a balanced one depend on the specific characteristics of each country, but also on regional characteristics. For example, the trends in mortality in the Baltic countries, Estonia, Latvia and Lithuania, are very similar (Kasmel et al. 2004). Therefore, it is not surprising to find similar gaps between the female and male timing for the crossing in life expectancies (Table 1 and Figure 3). Females from Estonia, were near to have a crossing in the 1970s closer to the situation for the other two countries. However, it is surprising to find these countries with the extreme of over 14 years of difference in the female and male timing, while other eastern European countries, except Bulgaria and Ukraine, only
differ by a couple of years. Sex differentials in infant mortality vary widely across countries (Fuse and Crenshaw 2006). However, in the context of the Baltic countries this is probably not the only factor. The great disparity between females and males comes from a combination of infant mortality and high levels of adult male mortality as indicated in equation (2). Furthermore, existent differentials among subpopulations within each country could also drive some of these results (Leinsalu et al. 2004).

This analysis can be considered as a within-population examination of infant mortality versus mortality after age one. However, the level of mortality of a population is a key element for comparison across populations or across time. As shown in Figure 5, the world is highly heterogeneous in terms of the current situation vis a vis balance or imbalance in life tables. We set a threshold that divides countries into those that have achieved a balanced situation, given by equation (2), and those that have not. This division is mainly due to the high levels of infant mortality still remaining in most of the world.

The within comparison emphasized earlier can also be used to contrast subgroup differences. Populations can be compared on how infant versus after one mortality have changed. The black population in the US lags behind its white counterpart not only on levels of survivorship but also on the timing of the crossover between life expectancies at birth and age one (see Figure 6). This is due partly to the excess infant mortality still prevalent among blacks as compared with other races (Martin et al. 2005). It is also a consequence of the mortality levels at adult ages which have progressed and regressed with unexpected fluctuations (Harper et al. 2007). For example, in recent years the reduction in young adult mortality among black males will cause a further delay in the life expectancy crossing if it is not accompanied with further reductions in infant mortality.

Research is needed into the factors that have produced all the above mentioned trends. Life table, the basic tool of demographic analysis, continues to yield insights in changes
in mortality, and could be of great help in assessing future questions of the crossover between life expectancies.

## Appendix

The relations in equations (2) and (3) can be derived by noting that life expectancy at birth is a function of the life expectancy at age one:

$$
\begin{equation*}
e_{0}(t)=e_{1}(t)\left[1-{ }_{1} d_{0}(t)\right]+{ }_{1} L_{0}(t) \tag{A1}
\end{equation*}
$$

Let $\Delta e_{0-1}(t)=e_{0}(t)-e_{1}(t)$ denote the difference in life expectancies at birth minus at age one at time $t$, derived from subtracting $e_{1}$ from equation (A1). To perfectly disentangle the share of the change that is due to mortality at young ages and the part due to mortality after age one its change over time is decomposed. The change over time in $\Delta e_{0-1}$ can be calculated and decomposed by calculating its derivative with respect to time as shown in equation (4). If data are available for two points in time, denoted as $t l$ and $t 2$, the relation in equation (4) can be estimated using Kitagawa's (1955) decomposition as

$$
\begin{equation*}
\dot{\Delta} e_{0-1}(t)=\left({ }_{1} L_{0}^{t 2}-{ }_{1} L_{0}^{t 1}\right)+\left({ }_{1} d_{0}^{t 1}-{ }_{1} d_{0}^{t 2}\right)\left[\frac{e_{1}^{t 1}+e_{1}^{t 2}}{2}\right]+\left(e_{1}^{t 1}-e_{1}^{t 2}\right)\left[\frac{{ }_{1} d_{0}^{t 1}+d_{1} d_{0}^{t 2}}{2}\right] \tag{A2}
\end{equation*}
$$

Crossings between life expectancies at birth and at other ages have also occurred over time. Similar to equations (2) to (4), it is possible to obtain relations between life expectancies at birth and at these older ages. For example, the crossing with life expectancy at age five occurs at the time when the inverse of the child mortality (mortality between birth and age five) is equal to life expectancy at age five

$$
\begin{equation*}
e_{0}(t)=e_{5}(t) \quad \Rightarrow \quad \frac{1}{{ }_{5} m_{0}}=e_{5}(t) \tag{A3}
\end{equation*}
$$

Figure 8 includes the timing of the crossing between $e_{0}$ and $e_{5}$ for the HMD countries with available data and the child mortality level at the time of the crossing.
[Here Figure 8]

Several countries from the HMD have mortality series that starts after the crossing between $e_{0}$ and $e_{5}$. For this reason we kept out of Figure 8: Slovenia, Germany (exGDR and ex-FRG), Estonia, Luxembourg, Latvia, Russia, Taiwan and Ukraine. New Zealand (total population) was also taken out for this reason, although the Non-Maori population from this country experienced the crossing in 1917. In Figure 8 it is more difficult to see regional differences, even when Eastern and Southern Europe achieve the crossing much later than Northern Europe. Portugal is the last country to achieve the equality in equation (A3) for the countries with available data in the HMD (2008).

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Table 1. Year, infant mortality, life expectancies at birth (e0) and age one (e1) at the time when they are equal, and the sex gap for the timing of the e0 and e1 crossing for all the countries/areas included in the HMD (2008).

|  |  | TO | POPUL | ATION |  | FEMALES |  |  | MALES |  | Diff. Males-Females |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 3-letter |  | Infant | Life |  | Infant | Life |  | Infant | Life |  |
| Country | code | Year | mortality | expectancy | Year | mortality | expectancy | Year | mortality | expectancy | Years |
| Australia | AUS | 1976 | 13.70 | 72.88 | 1974 | 13.76 | 75.46 | 1977 | 14.10 | 70.00 | 3 |
| Austria | AUT | 1980 | 14.64 | 72.70 | 1978 | 12.78 | 75.66 | 1982 | 15.21 | 69.38 | 4 |
| Belarus | BLR | 1985 | 14.20 | 70.86 | 1983 | 13.05 | 75.93 | 1988 | 15.55 | 67.24 | 5 |
| Belgium | BEL | 1977 | 13.66 | 72.80 | 1976 | 13.33 | 75.42 | 1979 | 13.57 | 69.92 | 3 |
| Bulgaria | BGR | 1988 | 13.60 | 71.40 | 1985 | 13.63 | 74.26 | 2003 | 14.50 | 68.91 | 18 |
| Canada | CAN | 1976 | 13.26 | 73.92 | 1975 | 12.76 | 77.23 | 1977 | 13.87 | 70.64 | 2 |
| Check Rep. | CZE | 1984 | 14.18 | 71.00 | 1979 | 13.33 | 74.35 | 1985 | 14.04 | 67.51 | 6 |
| Denmark | DNK | 1971 | 14.03 | 73.44 | 1966 | 14.47 | 74.78 | 1972 | 14.62 | 70.80 | 6 |
| England and Wales | ENW | 1977 | 13.55 | 73.47 | 1975 | 13.63 | 75.89 | 1980 | 13.76 | 70.74 | 5 |
| Estonia | EST | 1984 | 13.64 | 69.65 | 1983 | 13.05 | 74.83 | 1988 | 13.86 | 66.44 | 5 |
| Finland | FIN | 1968 | 13.89 | 69.79 | 1967 | 13.51 | 73.59 | 1970 | 14.98 | 66.16 | 3 |
| France | FRA | 1975 | 13.51 | 72.98 | 1973 | 13.57 | 76.31 | 1976 | 14.46 | 69.17 | 3 |
| Germany, East | GDR | 1978 | 13.59 | 72.00 | 1972 | 13.89 | 73.94 | 1980 | 14.35 | 68.71 | 8 |
| Germany, West | FRG | 1979 | 13.70 | 73.17 | 1977 | 13.45 | 75.85 | 1980 | 14.52 | 69.87 | 3 |
| Hungary | HUN | 1992 | 13.82 | 69.17 | 1990 | 13.44 | 73.79 | 1992 | 15.38 | 64.66 | 2 |
| Iceland | ISL | 1960 | 13.02 | 74.09 | 1957 | 12.63 | 75.16 | 1967 | 13.64 | 71.10 | 10 |
| Italy | ITA | 1982 | 13.05 | 74.98 | 1980 | 12.39 | 77.43 | 1982 | 14.48 | 71.60 | 2 |
| Japan | JPN | 1970 | 13.45 | 72.07 | 1968 | 13.10 | 74.29 | 1971 | 14.41 | 70.13 | 3 |

## Continue Table 1.

| Country | TOTAL POPULATION |  |  |  |  | FEMALES |  |  | MALES |  | Diff. Males-Females |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 3-letter code | Year | Infant mortality | Life expectancy | Year | Infant mortality | Life expectancy | Year | Infant mortality | Life expectancy | Years |
| Latvia | LVA | 1982 | 14.22 | 69.71 | 1970 | 13.81 | 74.24 | 1984 | 14.88 | 64.08 | 14 |
| Lithuania | LTU | 1983 | 14.51 | 70.98 | 1971 | 12.99 | 76.13 | 1986 | 13.71 | 67.92 | 15 |
| Luxembourg | LUX | 1972 | 13.48 | 70.71 | 1972 | 11.55 | 73.88 | 1977 | 14.33 | 67.94 | 5 |
| Netherlands | NLD | 1967 | 13.12 | 73.82 | 1963 | 13.30 | 75.77 | 1970 | 13.67 | 70.83 | 7 |
| New Zealand | NZL | 1977 | 14.09 | 72.22 | 1973 | 13.17 | 75.14 | 1980 | 13.92 | 70.04 | 7 |
| Norway | NOR | 1968 | 13.17 | 73.96 | 1965 | 14.20 | 76.52 | 1971 | 14.78 | 71.14 | 6 |
| Portugal | PRT | 1988 | 13.12 | 74.07 | 1987 | 12.75 | 77.40 | 1988 | 14.27 | 70.41 | 1 |
| Russia | RUS | 2001 | 15.05 | 65.29 | 2000 | 13.49 | 72.24 | 2001 | 17.52 | 58.89 | 1 |
| Slovak Rep. | SVK | 1987 | 13.95 | 71.08 | 1985 | 14.09 | 74.75 | 1989 | 14.70 | 66.80 | 4 |
| Slovenia | SVN | 1983 | 13.54 | 70.88 | 1983 | 12.72 | 74.88 | 1983 | 14.34 | 66.77 | 0 |
| Spain | ESP | 1979 | 14.28 | 75.12 | 1979 | 12.35 | 78.01 | 1980 | 13.57 | 72.31 | 1 |
| Sweden | SWE | 1965 | 13.40 | 73.88 | 1964 | 13.09 | 75.88 | 1969 | 13.67 | 71.73 | 5 |
| Switzerland | CHE | 1972 | 13.12 | 73.78 | 1970 | 13.10 | 76.16 | 1974 | 14.65 | 71.17 | 4 |
| Taiwan | TWN | 1973 | 14.37 | 69.45 | 1971 | 13.99 | 71.78 | 1973 | 15.40 | 67.07 | 2 |
| Ukraine | UKR | 1988 | 14.22 | 70.93 | 1971 | 13.28 | 74.76 | 1989 | 14.90 | 66.13 | 18 |
| United States | USA | 1979 | 13.48 | 74.03 | 1977 | 12.65 | 77.08 | 1980 | 14.22 | 69.99 | 3 |

Source: Human Mortality Database (2008).
Note. The year showed in this Table correspond to either the first year with a higher life expectancy at birth than at age one or the year before. The lowest difference between life expectancies at birth and at age one between these two years was assumed to be the likely year where the crossing occured. For some countries there were some years of fluctuations between the values of life expectancies at birth and at age one (higher and lower values of e0 respect to e1) before it actually turned into a balanced life table with $\mathrm{e} 0>\mathrm{e} 1$.

Figure 1. Life expectancy by age for the Icelandic total population in 1860, 1900, 1950 and


Source: Human Mortality Database (2008).

Figure 2. Life expectancies at birth ( $\mathrm{e}_{0}$ ) and age one ( $\mathrm{e}_{1}$ ) and the inverse of the infant mortality (1/1mo) for the Swedish total population from 1850 to 2000.


Source: Human Mortality Database (2008). Only values of inverse infant mortality below 90 are shown.

Figure 3. Life expectancy at birth (eo) at time of the first crossing with life expectancy at age one (e $\mathrm{e}_{1}$ ), total population by country.


[^0]Figure 4a. Components of change in the difference between male life expectancies at birth and at age one by decade, Japan 1960-2004.


Figure 4b. Components of change in the difference between male life expectancies at birth and at age one by decade, Russia 1960-2005.


Figure 4c. Components of change in the difference between male life expectancies at birth and at age one by decade, United States 1960-2003.


Figure 5a. Infant mortality rate and life expectancy at birth for countries of the world not included in the HMD, latest available information from WHO and UN.


Figure 5b. Model life tables and crossing in life expectancies, males.


Figure 6. Infant mortality and Life expectancy at birh for females and males by race (white \& black) in the United States, 1980 and 1982-2003.


Figure 7a. Components of change in the difference between male life expectancies at birth and at age one for the black American population by 5 years, 1980 to 2003.


Figure 7b. Components of change in the difference between male life expectancies at birth and at age one for the white American population by 5 years, 1980 to 2003.


Figure 8. Year and child mortality rate at time of the first crossing between life expectancy at birth and at age five, HMD 1910-1990.


[^1]
[^0]:    Source: Human Mortality Database (2008); Table 1 has the countries 3 -letter digits.

[^1]:    Source: Human Mortality Database (2008).

