Modelling and projecting the postponement of childbearing in low-fertility countries

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Abstract

In most developed countries, total fertility reached below-replacement level and stopped changing notably. Contrary to the almost stationary fertility level, the mean age of childbearing (*MAC*) is increasing markedly, reflecting a more general 'postponement transition' that may correspond to fundamental shifts for society (Lee and Goldstein, 2003, Billari, 2005). The postponement of childbearing, however, includes changes in fertility at all reproductive ages, which are more complicated than the increase of *MAC*. Using the idea of the Lee-Carter (1992) method that deals with mortality, we show that the changes in the age pattern of fertility can be well described by a single variable, which leads to simple ways of projection. We discuss some application issues using the data from Italy, and describe a condition under which the model is expected to work well for other countries.

The model and its estimations

Most fertility studies deal with age-specific fertility rate (*ASFR*), which sums to total fertility (*TF*) over the reproductive ages. To model the age pattern of fertility, we focus on the proportionate *ASFR*, which is *ASFR/TF*, and sums to 1 over the reproductive ages. Denote the proportionate *ASFR* at age x and time t by $Fp(x, t)$, and denote the over-time average of $Fp(x, t)$ by $a(x)$, the model can be written as

$$Fp(x,t) \approx a(x) + b(x)k(t) . \qquad (1)$$

where $b(x)$ is the rate of change by age groups and $k(t)$ is the overall time trend.
One may see immediately that the right-hand side is identical to the Lee-Carter (1992) model. In fact, what we really borrowed from the Lee-Carter method is the idea, which is

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† The views expressed in this paper are those of the authors and do not necessarily reflect those of the United Nations. Its contents has not been formally edited and cleared by the United Nations.

to convert the task of dealing with a group of variables (*Fp(x,t)* at different ages) to that of handling a single variable *k(t)*.

Parameters *b(x)* and variable *k(t)* in model (1) can be estimated in various ways, and we discuss two below.

The first estimation is obtained using the ordinary least squares (OLS). To see this, let

$$\sum_x xb(x) = 1. \qquad (2)$$

Thus, *k(t)* is the difference between the observed *MAC* at time *t* and the *MAC* given by *a(x)*:

$$\sum_x xFp(x,t) - \sum_x xa(x) = k(t)\sum_x xb(x) = k(t). \qquad (3)$$

Since *k(t)* is known, *b(x)* is solvable using OLS to minimise

$\sum_t \sum_x [Fp(x,t) - a(x) - b(x)k(t)]^2$ with constrain (2):

$$b(x) = \frac{\sum_t Fp(x,t)k(t)}{\sum_t k^2(t)}. \qquad (4)$$

An advantage of the first estimation is that the model *MAC* is identical to the observed value at any time. The disadvantage of first estimation is, however, that the difference between the observed and model *Fp(x,t)* has no constraint on valid bounds.

The second estimation is obtained using the singular value decomposition (SVD), which provides the values of *b(x)* and *k(t)* that minimises

$\sum_t \sum_x [Fp(x,t) - a(x) - b(x)k(t)]^2$ (see, Lee and Carter, 1992). For their convenience, Lee and Carter scaled the *b(x)* to sum to 1 over all *x*. For our convenience, we scale the *b(x)* as in (2), so that *k(t)* is the difference between the model *MAC* at time *t* and the *MAC* given by *a(x)*. The advantage of the second estimation is that the sum of the squared difference between the observed and model *Fp(x,t)* is minimised. A disadvantage of the second estimation, however, is that there are differences between the observed and model values of *MAC*, which is zero using the first estimation.

Some application issues

Denote by *Fpm(x,t)* the model value of *Fp(x,t)*. The explanation ratio, which indicates the proportion of the variance of *Fp(x,t)* explained by *Fpm(x,t)*, is defined as

$$R = \frac{\sum_t \sum_x [Fp(x,t) - Fpm(x,t)]^2}{\sum_t \sum_x [Fp(x,t) - a(x)]^2}. \quad (5)$$

Using the data on *Fp(x,t)* in Table 1, we have *R=0.85* (SVD) and *R=0.75* (OLS).

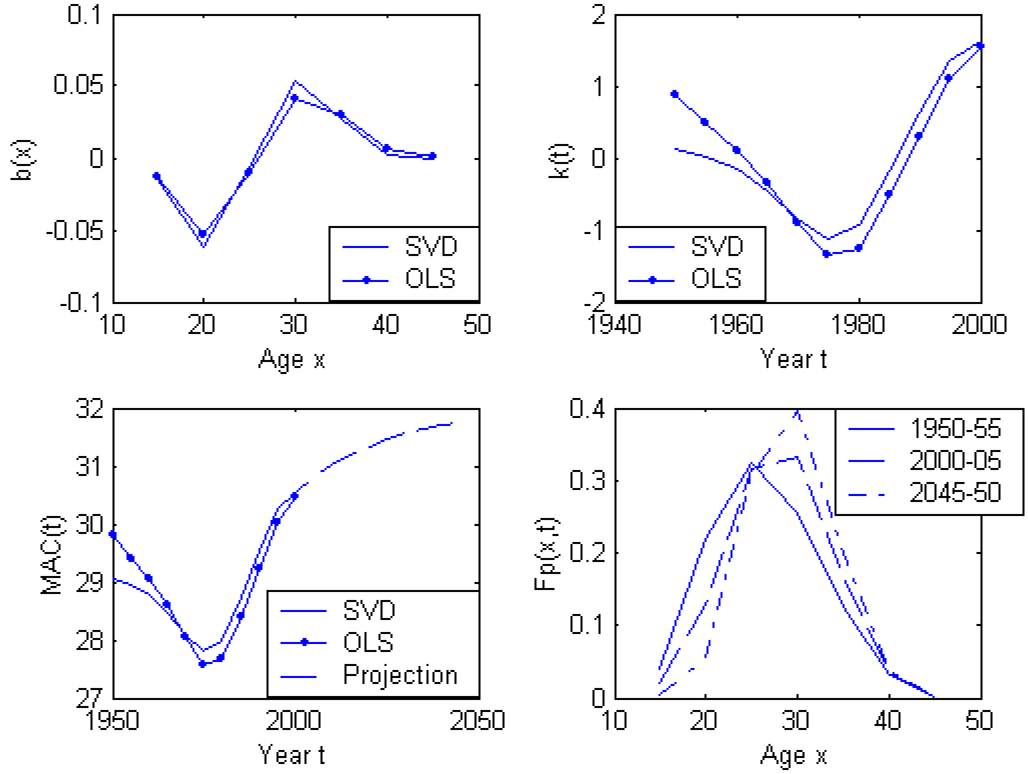
Table 1. Italian proportionate *ASFR*

	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1950-1955	0.033	0.215	0.301	0.226	0.162	0.057	0.005
1955-1960	0.037	0.224	0.311	0.232	0.142	0.048	0.004
1960-1965	0.041	0.234	0.322	0.231	0.124	0.044	0.003
1965-1970	0.050	0.257	0.317	0.220	0.117	0.036	0.003
1970-1975	0.063	0.278	0.322	0.201	0.103	0.032	0.002
1975-1980	0.069	0.300	0.323	0.194	0.086	0.025	0.003
1980-1985	0.055	0.288	0.347	0.208	0.084	0.018	0.001
1985-1990	0.038	0.237	0.363	0.248	0.096	0.019	0.000
1990-1995	0.031	0.185	0.354	0.289	0.119	0.022	0.000
1995-2000	0.029	0.144	0.327	0.326	0.147	0.026	0.002
2000-2005	0.028	0.134	0.295	0.337	0.172	0.034	0.000

Sources: UNSD and Eurostat

Thus, as we expect, SVD worked better than OLS did in terms of describing *Fp(x,t)*. On the other hand, as is shown in the third panel of Figure 1, SVD cannot perfectly fit the *MAC(t)*, which is described exactly by OLS. Therefore, which estimation to use depends on which variable, *Fp(x,t)* or *Mac(t)*, is more important to the user.

Figure 1. Results from using Italian data in 1950-2005



Why does the model work well? Putting in the continuous version, (1) yields

$$\frac{1}{[Fp(x,t)-a(x)]}\frac{d}{dt}[Fp(x,t)-a(x)] \approx \frac{1}{k(t)}\frac{d}{dt}k(t),$$
$$\frac{\partial}{\partial x}\{\frac{1}{[Fp(x,t)-a(x)]}\frac{d}{dt}[Fp(x,t)-a(x)]\} \approx 0. \qquad (6)$$

Thus, the condition for (1) to work well is that the rates of change in *[Fp(x,t)-a(x)]* are similar at different ages. In fact, the changes of age pattern of fertility that we concern are two rotations. The first rotation is caused by the reduction of childbearing at older ages, in which *[Fp(x,t)-a(x)]* rises at younger and drops at older ages. And the second rotation is due to postponing childbearing, in which *[Fp(x,t)-a(x)]* drops at younger and rises at older ages. When such rotations take place at the rates that are similar over ages, (6) holds. Therefore, we may expect (1) to work well not only for the data of Italy, but also those when the rotations take place evenly over age.

Turning to projection, the above model will yield a maximum *MAC*, namely *MACm*, older than which the model will produce negative values for *Fp(x,t)* at some younger ages. This is because that *b(x)* is negative at younger ages, which will make *Fp(x,t)* negative when *k(t)* rises to a certain level *Km*:

$$Km = \min[-\frac{a(x)}{b(x)}], \quad b(x) < 0\,. \qquad (7)$$

Since *k(t)* differs with *MAC(t)* by a constant $\sum_x xa(x)$, *Km* leads to the *MACm*. According to (1) and (2), the maximum *MAC* is written as

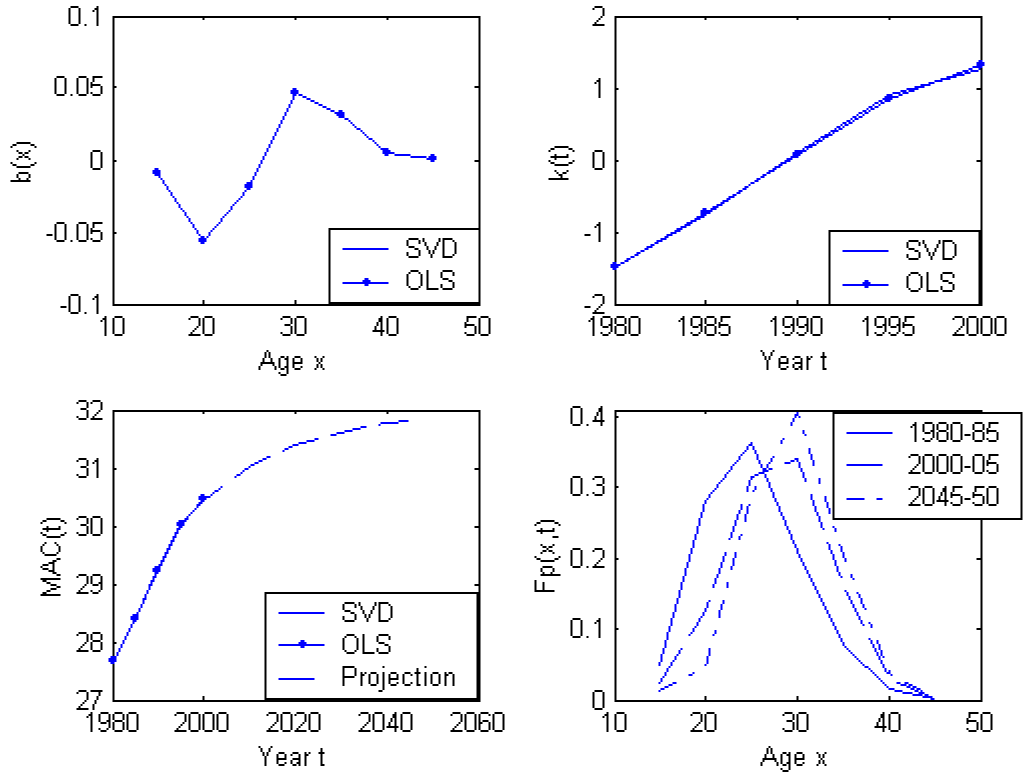
$$MACm = \sum_x xa(x) + Km\,. \qquad (8)$$

Using data in 1950-2005, the *MACm* is estimated as 32.01 by SVD, or 32.21 by OLS.

Given the model (SVD) values of *MAC(t)* and *MACm* as example, there are various simple ways to project future *MAC(t)*. Among these, the one shown in the third panel is perhaps the simplest, in which the *MAC(t)* converges to *MACm* exponentially at the pace measured from its last two values. When *MAC(t)* is projected, so are the *Fp(x,t)* using (1), as can be seen in the last panel of Figure 1.

When the *MACm* given by (8) looks too small to be plausible, for example of many East European countries, we suggest use a plausible maximum *MAC* that may be taken from other countries, and replace the subsequent negative values of the projected *Fp(x,t)* at a certain *x* by its last positive value. By doing so, we projected a more plausible trajectory for the *MAC*, and stopped following the modelled projection of *Fp(x,t)* when it becomes negative.

Figure 2. Results from using Italian data in 1980-2005



References

Lee, R. D. and L. Carter. 1992. "Modelling and Forecasting the Time Series of U.S. Mortality." *Journal of the American Statistical Association* 87: 659—71.

Lee, R. and J.R. Goldstein. 2003. Rescaling the life cycle: Longevity and proportionality. Population and Development Review, 29: 183-207.

Billari, F. 2005. Partnership, childbearing and parenting: Trends of the 1990s. In The new demographic regime: Population challenges and policy responses. United Nations: New York and Geneva.

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Session 164: Timing of fertility and family transitions in Europe

United Nations Population Division
Estimates and Projection Section

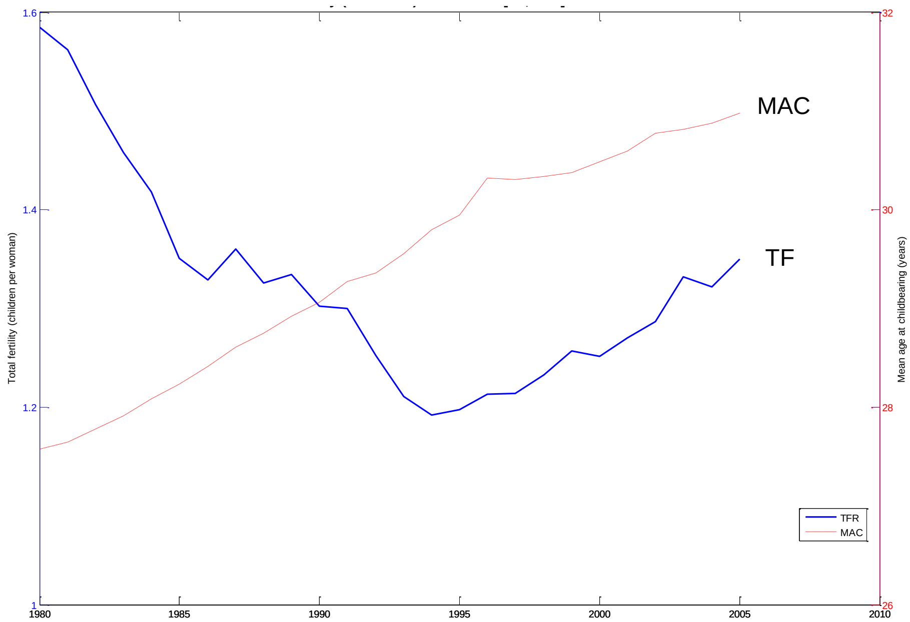
Outline

- Background
- Data
- Estimation and forecasting model
- Results and findings

1. **Issue**: Total fertility vs. Mean Age of Childbearing (1980-2007)

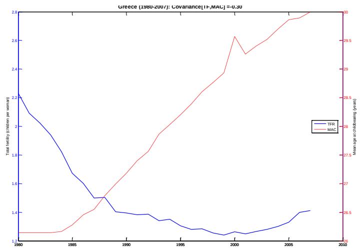
MAC keeps rising while TF stagnates below replacement level or fluctuates toward replacement level

Italy: Covariance [TF,MAC]=**-0.09**

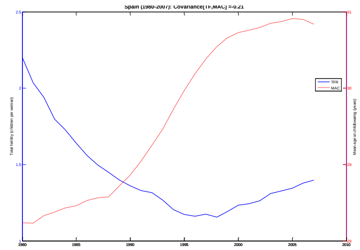


Southern Europe

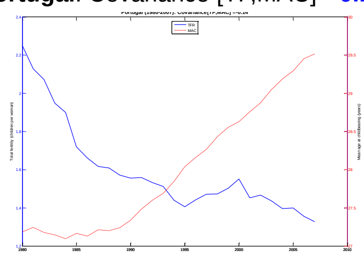
Greece: Covariance [TF,MAC]=**-0.30**

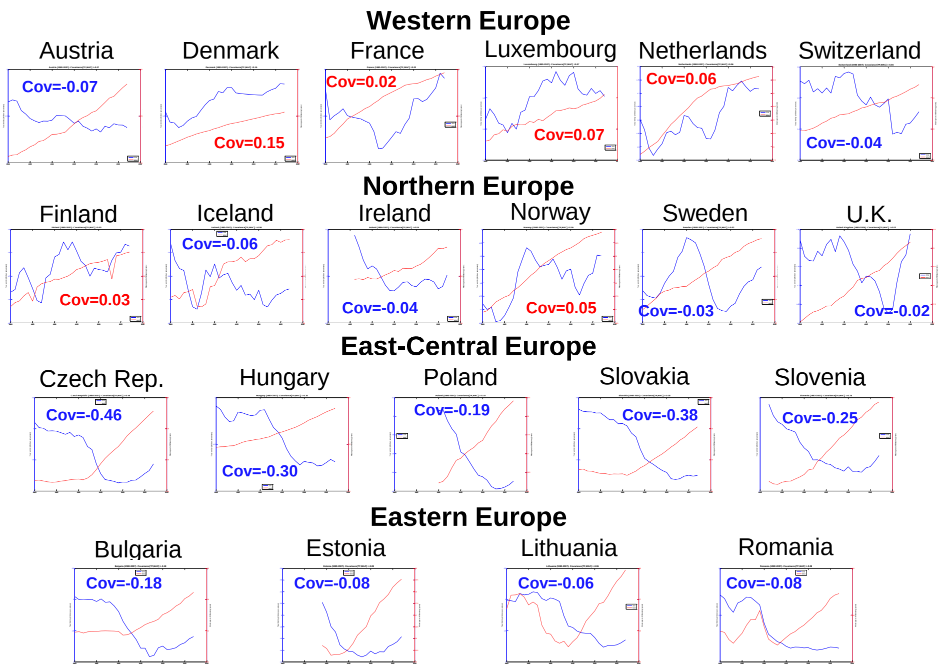


Spain: Covariance [TF,MAC]=**-0.21**



Portugal: Covariance [TF,MAC]=**-0.14**





2. Data

Data sources

- Eurostat database (http://europa.eu/estatref/download/everybody)
- 25 countries with annual fertility rates by single age for the last 15 years or more
  1. **Western Europe**: Austria, Denmark, France métropolitaine, Luxembourg (Grand-Duché), Netherlands, Switzerland
  2. **Northern Europe**: Finland, Iceland, Ireland, Norway, Sweden, United Kingdom
  3. **Southern Europe**: Greece, Italy, Portugal, Spain
  4. **East-Central Europe**: Czech Republic, Hungary, Poland, Slovakia, Slovenia
  5. **Eastern Europe**: Bulgaria, Estonia, Lithuania, Romania

3. Model and estimation strategy

Age pattern of fertility modelling

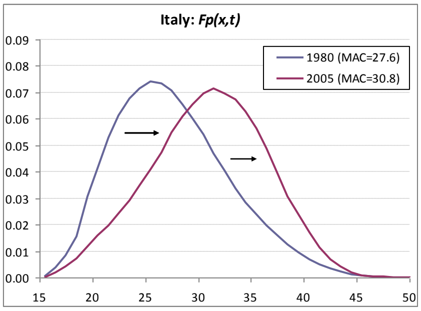
F(x,t) = age-specific fertility rate at age x and time t

$$TF(t) = \sum_{x=15}^{50} F(x,t)$$

***Fp(x,t)* = proportionate age-specific fertility rate at age x and time t**

$$Fp(x,t) = \frac{F(x,t)}{TF(t)}$$

$$\text{with } \sum_{x=15}^{50} Fp(x,t) = 1$$



Age pattern of fertility modelling

$$Fp(x,t) \approx a(x) + b(x)k(t)$$

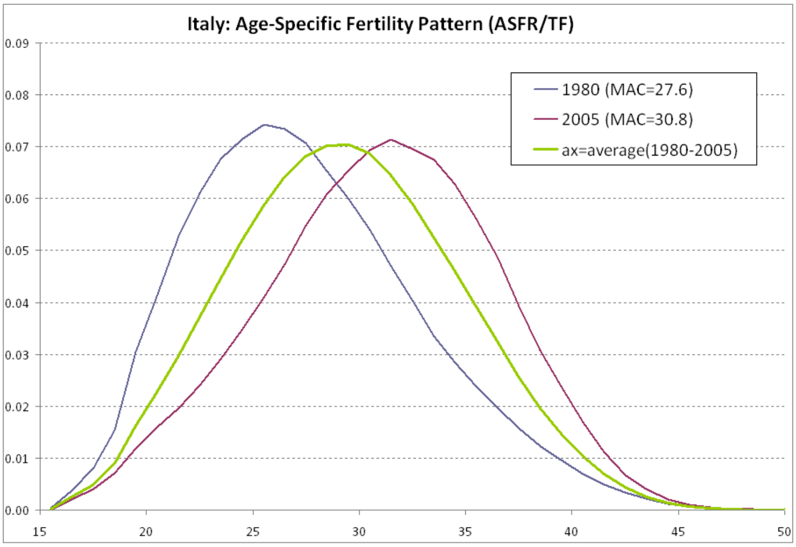
Identical to Lee, R. D. and L. Carter. 1992. “Modelling and Forecasting the Time Series of U.S. Mortality.” *Journal of the American Statistical Association* 87: 659—71.

a(x) = over-time average of *Fp(x, t)*

b(x) = rate of change by age groups

k(t) = overall time trend

a(x) = average (past) age pattern



Two ways to estimate *b(x)* and *k(t)*

(1) Ordinary Least Squares (OLS)

$$rescaling \quad b(x) \ to \ get \ \sum_x xb(x) = 1$$

k(t) = difference between observed *MAC* at time *t* and *MAC* given by *a(x)*

$$\sum_x xFp(x,t) - \sum_x xa(x) = k(t)\sum_x xb(x) = k(t)$$

$$and \quad b(x) = \frac{\sum_t Fp(x,t)k(t)}{\sum_t k^2(t)}$$

Solved by minimizing

$$\sum_t \sum_x [Fp(x,t) - a(x) - b(x)k(t)]^2$$

under constraint that $\sum_x xb(x) = 1$

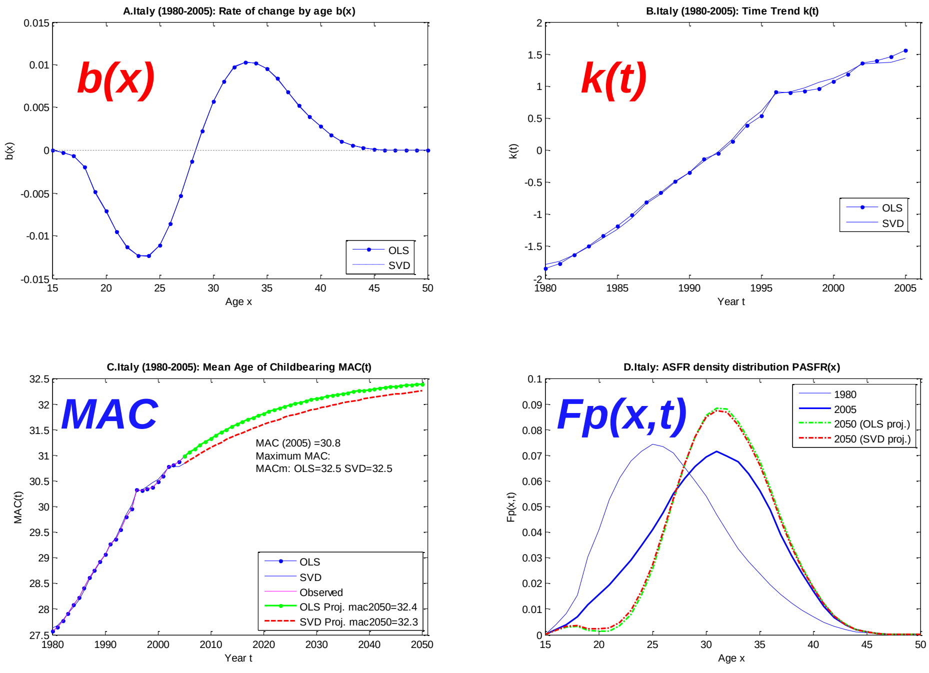
Two ways to estimate b(x) and k(t)

(2) Singular Value Decomposition (SVD) by minimizing:

$$\sum_t \sum_x [Fp(x,t) - a(x) - b(x)k(t)]^2$$

$$rescaling \quad b(x) \ to \ get \ \sum_x xb(x) = 1$$

k(t) = difference between the model *MAC* at time *t* and the *MAC* given by *a(x)*



OLS vs. SVD: Pros and cons

(1) **OLS**: best to fit MAC, but estimates of b(x) and k(t) done sequentially, and difference between the observed and model *Fp(x,t)* has no constraint on valid bounds

(2) **SVD**: best to fit overall age pattern, estimates of b(x) and k(t) done simultaneously with constraints on valid bounds – but differences between observed and model values of *MAC*

Goodness of fit

Explanation Ratio (R) = proportion of the variance of *Fp(x,t)* explained by *Fpm(x,t)* which is the model value of *Fp(x,t)*

$$R = \frac{\sum_t \sum_x [Fp(x,t) - Fpm(x,t)]^2}{\sum_t \sum_x [Fp(x,t) - a(x)]^2}$$

Maximum MAC

MACm = age limit at which the model will produce negative values for *Fp(x,t)* at younger ages because *b(x)* < 0 at younger ages and *k(t)* rises to a certain level *Km:*

$$Km = \min[-\frac{a(x)}{b(x)}], \quad b(x) < 0$$

Since *k(t)* differs with *MAC(t)* by a constant $\sum_x xa(x)$ *Km* leads to…

$$MACm = \sum_x xa(x) + Km$$

K(t) projection from 2005-07 to 2050

Only *k(t)* term in model is time dependent and needs to be projected…

Since k(t) = difference between the model *MAC* at time *t* and the *MAC* given by *a(x)*

→ Project *MAC(t)* to converge toward *MACm* exponentially using trend for past 10 years.

4. Results and Findings

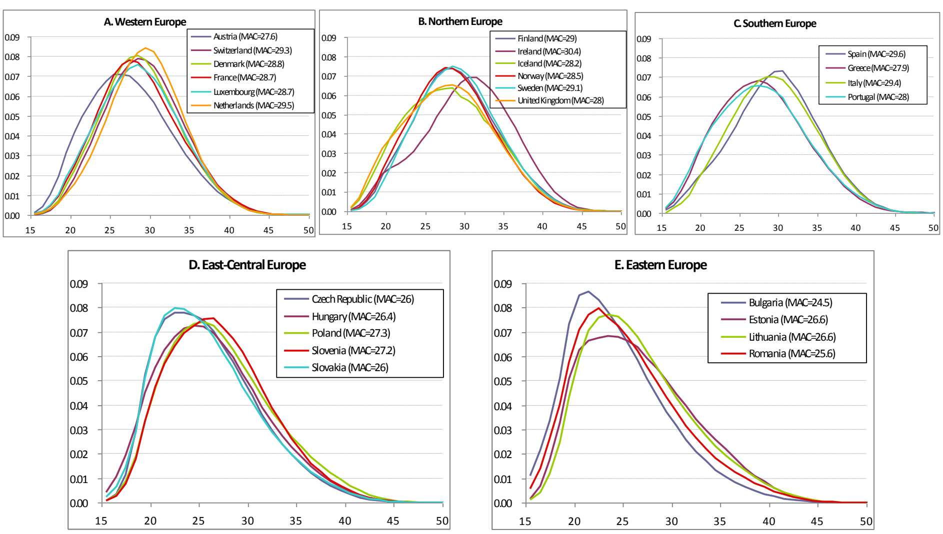
Model fitting performance

(1) very good fit for most countries (>80% or even 90% variance explained)

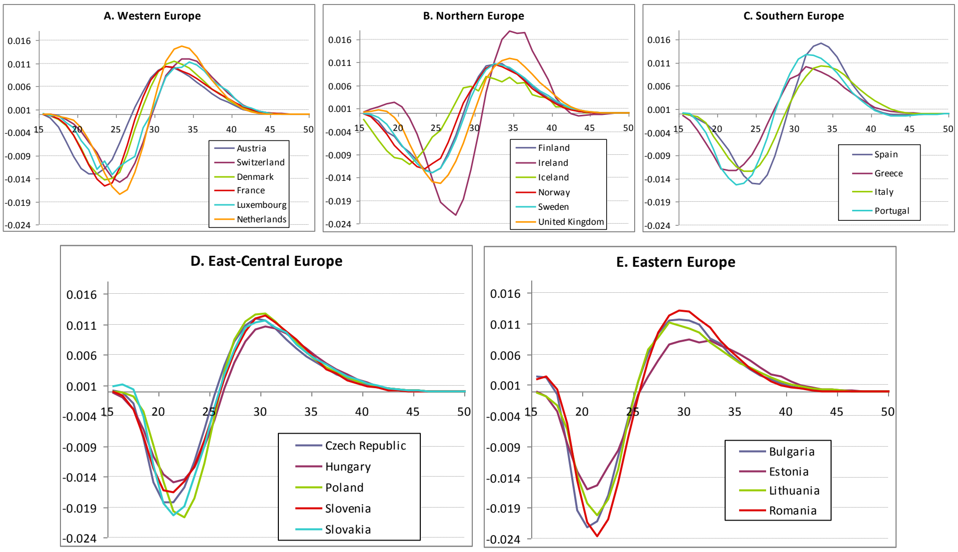
(2) SVD fit outperforms OLS fit

Country	minYear	maxYear	CovTfMac	R(OLS)	R(SVD)
Western Europe					
Austria	1980	2007	-0.07	0.93	0.93
Denmark	1980	2007	0.15	0.96	0.97
France	1980	2007	0.02	0.94	0.95
Luxembourg	1980	2007	0.07	0.79	0.80
Netherlands	1980	2007	0.06	0.95	0.96
Switzerland	1980	2007	-0.04	0.91	0.92
Northern Europe					
Finland	1980	2007	0.03	0.88	0.89
Iceland	1980	2007	-0.06	0.81	0.82
Ireland	1986	2007	-0.04	0.85	0.89
Norway	1980	2007	0.05	0.97	0.97
Sweden	1980	2007	-0.03	0.96	0.97
United Kingdom	1980	2005	-0.02	0.94	0.96
Southern Europe					
Greece	1980	2007	-0.30	0.91	0.91
Italy	1980	2005	-0.09	0.93	0.93
Portugal	1980	2007	-0.14	0.89	0.90
Spain	1980	2007	-0.21	0.85	0.86
East-Central Europe					
Czech Republic	1980	2007	-0.46	0.95	0.95
Hungary	1980	2007	-0.30	0.93	0.93
Poland	1990	2007	-0.19	0.97	0.98
Slovenia	1982	2007	-0.25	0.93	0.93
Slovakia	1980	2007	-0.38	0.96	0.96
Eastern Europe					
Bulgaria	1980	2007	-0.18	0.92	0.95
Estonia	1989	2007	-0.08	0.95	0.95
Lithuania	1980	2007	-0.06	0.70	0.77
Romania	1980	2007	-0.08	0.81	0.89

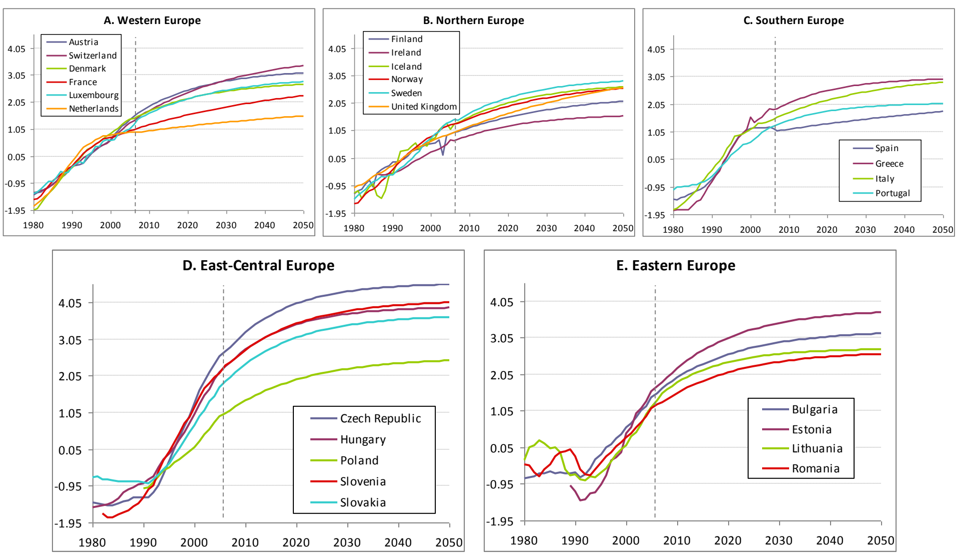
a(x) = average age pattern of fertility



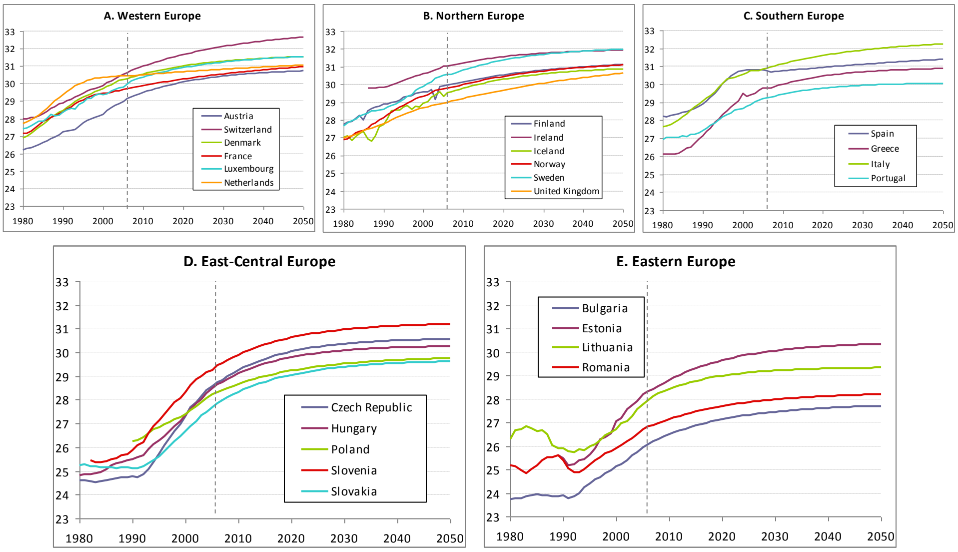
b(x) = rate of change by age



k(t) = overall time trend



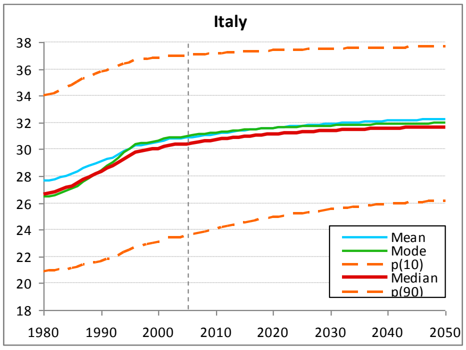
Mean Age of Childbearing *MAC(t)*



Mean Age of Childbearing

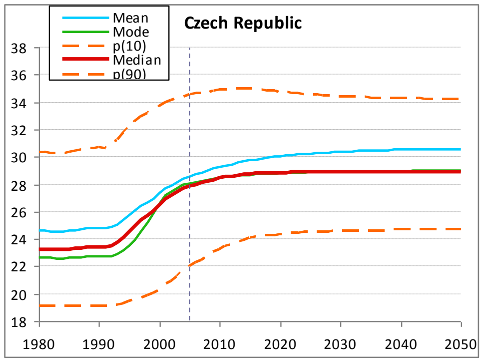
Country	Maximum MAC	Mean Age 2005	Mean Age 2050	Decennial Increase	Modal Age 2005	Modal Age 2050	Median Age 2005	Median Age 2050
Western Europe								
Austria	30.9	29.0	30.7	0.4	28.7	30.1	28.4	30.0
Denmark	31.7	30.3	31.5	0.3	29.9	30.8	29.6	30.8
France	31.7	29.7	31.0	0.3	29.1	30.1	29.0	30.2
Luxembourg	31.7	29.8	31.5	0.4	30.0	31.9	29.3	31.2
Netherlands	31.7	30.6	31.1	0.1	30.8	31.4	30.1	30.7
Switzerland	33.2	30.6	32.7	0.5	30.8	32.5	30.1	32.1
Northern Europe								
Finland	31.4	29.9	31.1	0.3	29.7	30.9	29.4	30.5
Iceland	31.0	29.4	30.9	0.3	28.8	29.9	28.6	30.0
Ireland	32.0	31.2	31.9	0.2	32.8	33.8	31.3	32.5
Norway	31.4	29.8	31.1	0.3	29.7	30.7	29.2	30.5
Sweden	32.1	30.5	32.0	0.3	30.5	31.5	30.0	31.3
United Kingdom	31.8	29.2	30.6	0.3	30.4	32.6	28.7	31.1
Southern Europe								
Greece	30.9	29.9	30.9	0.2	29.6	30.3	29.2	30.2
Italy	32.5	31.0	32.3	0.3	31.0	32.0	30.4	31.7
Portugal	30.1	29.3	30.1	0.2	29.7	30.5	28.9	29.8
Spain	32.4	30.9	31.4	0.1	31.5	32.0	30.7	31.2
East-Central Europe								
Czech Republic	30.6	28.6	30.6	0.4	28.1	29.0	27.9	28.9
Hungary	30.3	28.5	30.3	0.4	28.5	29.5	28.0	29.5
Poland	29.8	28.2	29.8	0.3	27.4	28.9	27.4	28.9
Slovenia	31.3	29.4	31.2	0.4	28.6	29.5	28.5	29.3
Slovakia	29.5	27.7	29.6	0.4	27.4	29.0	27.0	29.0
Eastern Europe								
Bulgaria	27.8	26.1	27.7	0.4	25.5	28.0	25.2	27.4
Estonia	30.0	28.2	30.3	0.5	27.3	29.1	27.4	29.5
Lithuania	29.3	27.6	29.3	0.4	26.3	28.0	26.7	28.3
Romania	28.3	26.7	28.2	0.3	26.3	28.5	26.0	27.9

Dispersion of the fertility age distributions

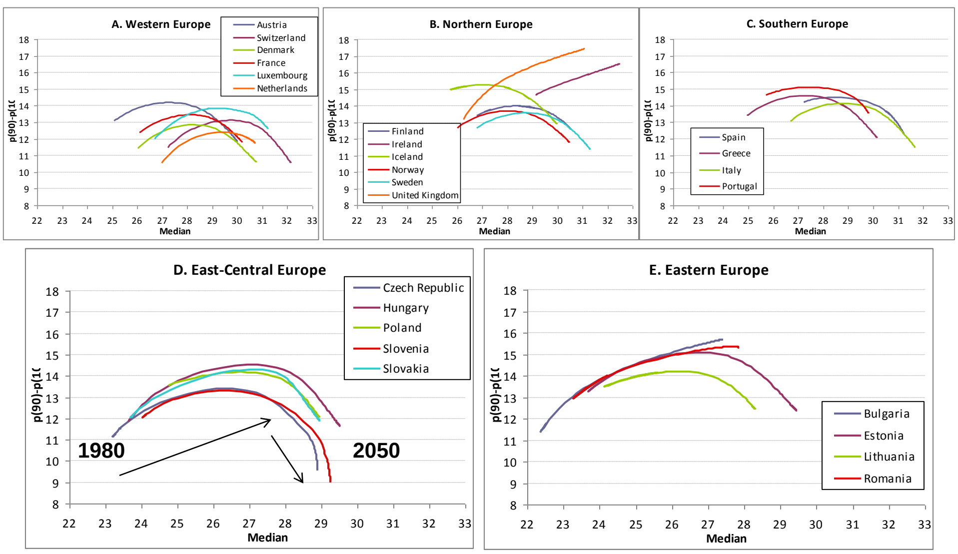


p(90) = Age for upper 90 percentile

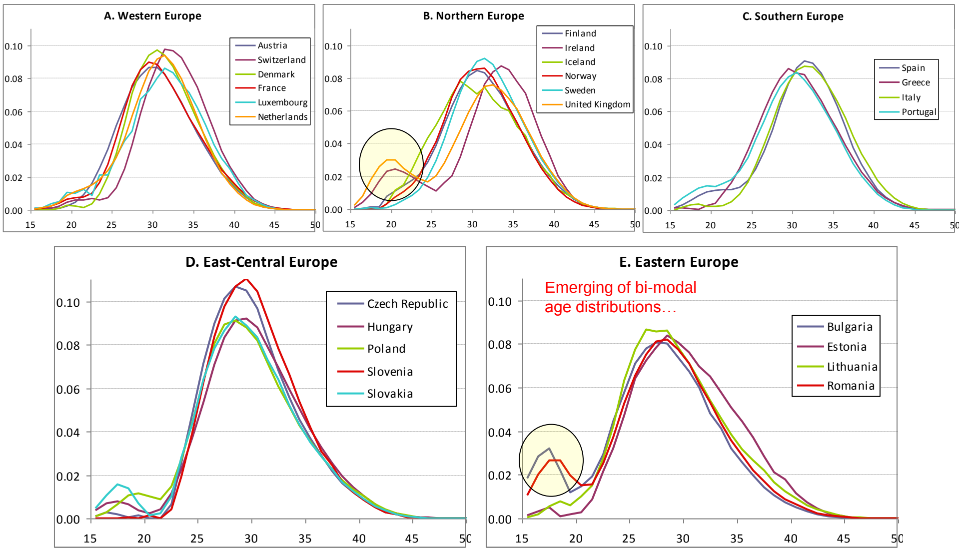
p(10) = Age for lower 10 percentile

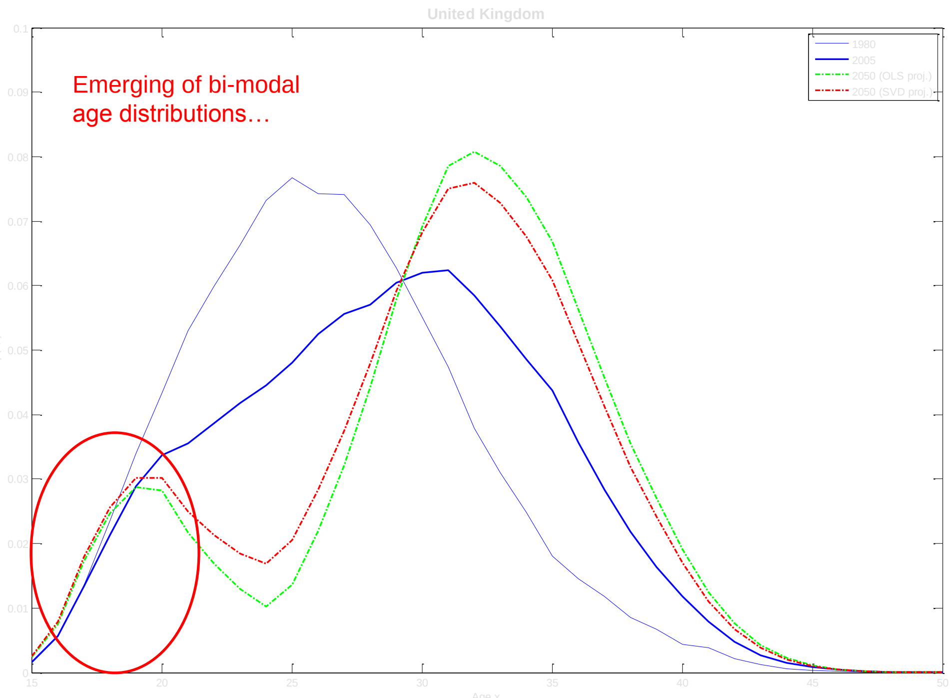


Median age vs. p(90)-p(10) age dispersion

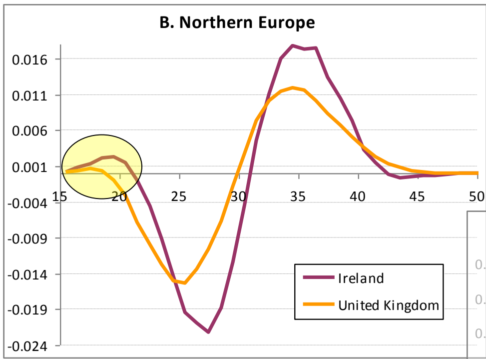


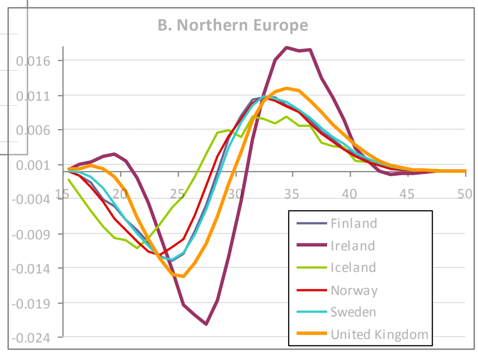
Projected age pattern in 2050





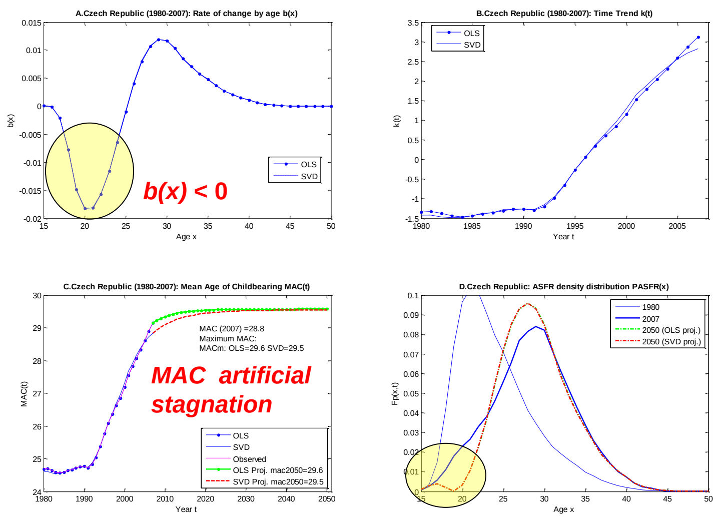
b(x) = rate of change by age





Lack of decline (or even increase) in fertility under age 20… in UK, Ireland, Slovakia, Bulgaria, Romania, etc.

Challenges with East-Central Europe and Eastern Europe

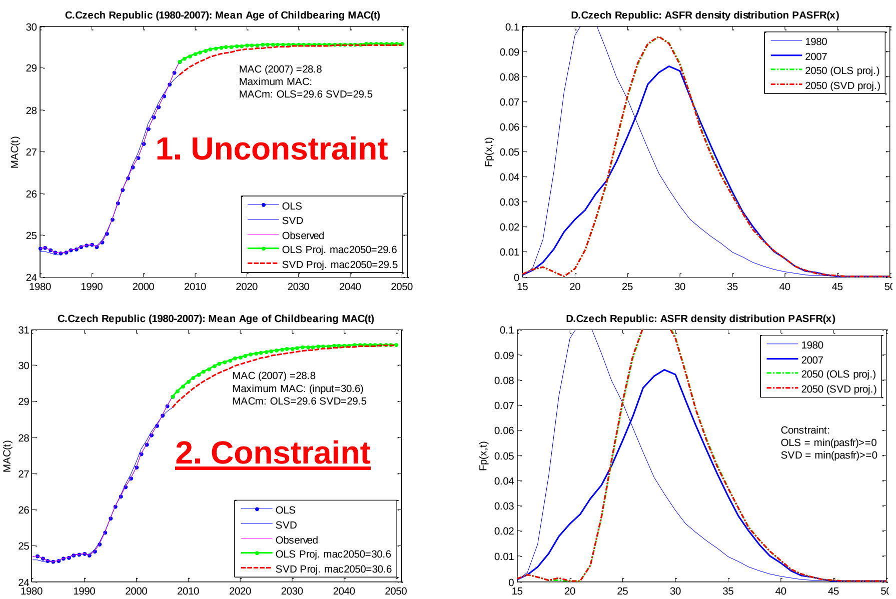


Fp(x,t) < 0
due to b(x) < 0

Additional constraints…

- If *MACm* <= latest observed MAC or implausibly low, compute *MACm* using experience from other countries… *MACm'* = latest MAC * ratio from all countries (except Central & Eastern Europe) of [average(*MACm*) / average(latest MAC)] = latest MAC * 1.05
- Replace subsequent negative values of the projected *Fp(x,t)* at a certain *x* by its last positive value.

Example: Czech Republic

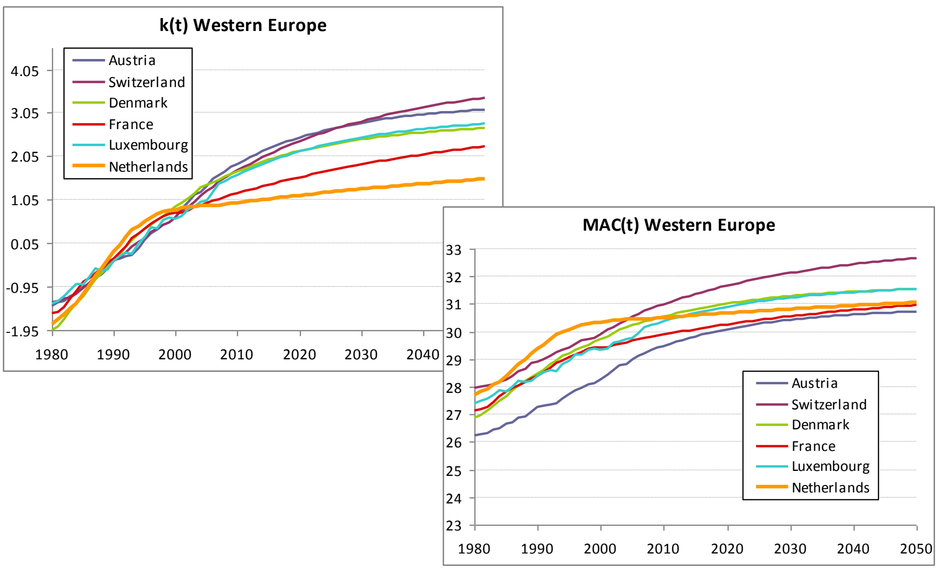


Sensitivity analysis

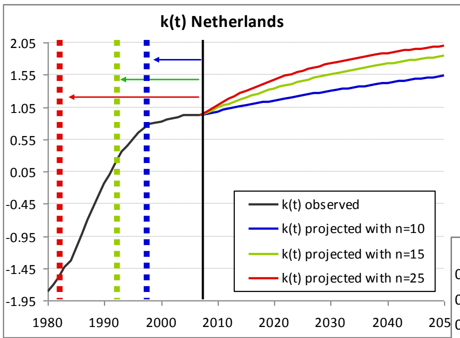
- How much does the length of the reference period matters to project *k(t)*?
- -> use the latest 10, 15,…, 25 years?

Country	Mean age in 2050 n=10	n=15	n=25
Western Europe			
Austria	30.7	30.7	30.6
Denmark	31.5	31.5	31.5
France	31.0	31.1	31.2
Luxembourg	31.5	31.5	31.4
Netherlands	**31.1**	**31.4**	**31.5**
Switzerland	32.7	32.6	32.5
Northern Europe			
Finland	31.1	31.1	31.1
Iceland	30.9	30.7	30.8
Ireland	31.9	31.9	
Norway	31.1	31.1	31.1
Sweden	32.0	32.0	31.8
United Kingdom	30.6	30.7	30.7
Southern Europe			
Greece	30.9	30.9	30.9
Italy	32.3	32.3	32.3
Portugal	30.1	30.1	30.0
Spain	31.4	32.1	32.0
East-Central Europe			
Czech Republic	30.6	30.5	30.4
Hungary	30.3	30.2	30.1
Poland	29.8	29.7	
Slovenia	31.2	31.2	31.1
Slovakia	29.6	29.6	29.4
Eastern Europe			
Bulgaria	27.7	27.7	27.5
Estonia	30.3	30.3	
Lithuania	29.3	29.3	29.0
Romania	28.2	28.2	27.9

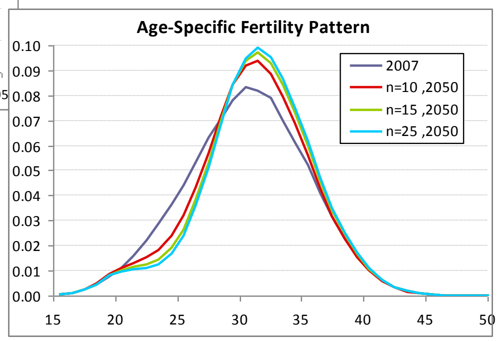
Netherlands: trend in MAC



Netherlands: trend in k(t)



Sensitivity of the length of period of reference used to project trend in k(t)… n= 10, 15, …, 25 on age pattern of fertility in 2050
→ Limited effect, overall very robust



Conclusion

- Overall approach robust and flexible to project fertility age patterns for below-replacement countries
- SVD fitting performs better than OLS to find b(x) and k(t)
- High consistency within and between regions for future age patterns of fertility
- Identification of special cases:
  1. Adolescent fertility not declining in some countries potentially leading to future bi-modal age distributions ;
  2. Special case of Central and Eastern Europe postponing childbearing through very pronounced declines at younger and rises at older ages requiring additional projection constraints.